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MAIN PRINCIPLES AND METHODS OF COMPLEX OPTIMISATION OF OPERATION CONTROL OF INTERCONNECTED POWER SYSTEMS AT THE INTERSTATE LEVEL IN FREE MARKET CONDITIONS

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Complex optimisation of the control of interconnected power systems at the interstate level is related to solution and correction of multi-level hierarchical problems of dynamic programming in general, its separate sub-problems differing in the time-space and situation aspects. Starting from the complex control of instantaneous steady-states (ESS), general principles of mathematical methods and tools, applied for solution of the multi-stage hierarchical dynamic programming problem, are considered.

Introduction

The previous paper of the authors in this edition and also the paper presented at the conference on operational research [1] give a short overview of the development of the theory and methods of complex optimisation of control and operational reliability of interconnected power systems (IPS) on the interstate level in free market conditions. The main topic of this overview – main principles and methods of complex optimisation – is considered more detailed in this paper, based on a more detailed report [2] by L. Krumm at the All-Russian scientific seminar with participation of foreign scientists "*Methodical problems of the reliability research of large energy systems*"**,** in Pskov on July 2–6, 2006, on the problems of reliability of the restructuring power systems.

Below the main principles and approaches of the developed methods are presented.

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First, the general description of the problem of complex optimisation of ESS of IPS is stated, on the example of instantaneous ESS. Mathematical principles of the elaborated and developed methods are presented.

Second, the main methodical principles (or approaches) of the analysis and optimisation of the control over IPS operation and their development at the interstate level are given.

Set-up of the problems of generalisation and development of the Generalised Method of Reduced Gradient (GMRG) for multi-objective correction of ESS, Pareto-optimal correction in particular, basing on the example of instantaneous ESS of IPS in market conditions

In the conditions of planned economy, the unique principle of optimisation of ESS of the United Electrical Power System (UEPS) at all levels of the dispatch control was commonly accepted. This principle was based on minimisation of the costs of electricity production and transport in the UEPS of the former USSR as a whole, i.e. with maximum benefit for the state. As the main criteria of the optimal planning and short-time control, the minimum costs of fuel were used.

In market conditions this principle could be applied to optimise the operation of separate independent EPS by some rational boundary parameters characterizing the tariffs of active and reactive energy sold, various reserves, controlled boundary parameters between EPS of different countries (like interchanged active and reactive power, voltage levels, etc.) and also parameters that determine appropriate configurations of the network.

Those rational boundary variables are determined by the market mechanism on N partners.

We shall consider a certain state of IPS (not necessarily optimal) the equilibrium state in the cooperation of the partners, if this state is determined by rational boundary variables within the present market mechanism.

However, the search for a rational version of boundary variables is still quite arbitrary, basing on the extremely simplified models with partial application of the optimisation apparatus, and a complex approach is not to be considered.

Despite of that, it could be proved that partners are striving towards maximisation of their incomes. In addition, it is clear that in the case of IPS *Baltija* we have to deal with three objective functions and not just with one.

In the short-term as well as in the long-term cases, different economic (income, in particular) and technical (losses or voltage deviations) indicators could serve as objective functions, depending on different situations.

In all cases, the improvement of ESS of IPS *Baltija* in the interests of all its partners remains a topical issue.

Striving for such improvement in the interests of all partners is initiated by the fact that the above market mechanism does not provide the equilibrium state being in the corresponding Pareto-optimal region* connected with application of one or the other type of objective functions. Moreover, this equilibrium state could, depending on the different situation, deviate from this domain considerably.

While changing all controls of IPS *Baltija* from some initial equilibrium state to some point in the Pareto space, a certain improvement of the state of all partners in this space by their objective functions occurs.

For rational improvement of ESS of IPS *Baltija* in the interests of all partners, it is expedient that the multi-objective approach will be used, in the case of which for all controls the minimum values from the set of functions will be maximised for all partners. If the improvement of the solution for an one-criterion sub-problem in the complex optimisation, using special mathematical tools for acceleration of the computing process GMRG, is called *complex optimal correction of ESS of EPS*, the improvement of the solution of multi-objective sub-problems of the complex optimisation using special mathematical tools for acceleration of the computing process [application instead of the gradient of the one implicit function the array of the multi-objective descent or rise (dependent on the minimisation or maximisation of the partners objective functions)], is called *complex multi-objective or Pareto-optimal correction of ESS of IPS*. With this approach, the characteristic of the techno-economical set-up of sub-problems of the complex Pareto-optimal corrections of instantaneous EES, and steady-states (ESS) of IPS, will have the following form [3, 4]:

Let the IPS of N independent states with indexes $s, q \in \mathbf{I}^R = (1,...,N)$ consist of a set of knots S_0, in this case this set could be factorised into corresponding subsets $\left\{ S_{0s} \,\middle|\, s \in \mathbf{I}^R \right\}$ of IPS of N independent states, whereby $S_0 = \underset{s \in \mathbf{I}^R}{U} S_{0s}$.

The set-up of the sub-problem of Pareto-optimal ESS IPS complex correction for instantaneous states in normal conditions, in the scope of main clauses of GMRG and considering the first main clause, in particular at application of the theory of implicit functions by continuous idealisation of changes of variables, leads to the following multi-objective search:

$$\min_Y F_s(\mathbf{Y}) \quad \mathit{or} \quad \max_Y F_s(\mathbf{Y}), \quad \forall\, s \in I^R, \tag{1}$$

where

$$F_s(\mathbf{Y}) = F_s\left[\mathbf{X}(\mathbf{Y},\mathbf{D}),\mathbf{Y},\mathbf{D}\right], \tag{2}$$

* Pareto-optimal region (domain) or just Pareto-domain is a space in the coordinates of control variables, such that in any point of this space, in the case of deviation, it is not possible to improve the state of all partners simultaneously by their objective functions.

i.e. to the search of a point $\mathbf{Y}_{\text{par}}^{\text{opt}}$ of the Pareto-optimal domain S_{par}^{opt} of the objective functions $\left\{F_s\left(\mathbf{Y}\right) \middle| s \in \mathbf{I}^R\right\}$, proceeding from the initial point $\mathbf{Y}_0$ of the equilibrium state of IPS (established by the market mechanism) with limitations

$$\mathbf{X}_{\mathbf{min}} \leq \mathbf{X}\left(\mathbf{Y}, \mathbf{D}\right) \leq \mathbf{X}_{\mathbf{max}}, \tag{3}$$

$$\mathbf{Y}_{\min} \leq \mathbf{Y} \leq \mathbf{Y}_{\max}, \tag{4}$$

where $F_s(\mathbf{Y})$, $\forall s \in \mathbf{I}^R$ some technical or economical parameter of the EPS s as an implicit objective function (2) of the array of independent variables $\mathbf{Y}$ of the whole IPS, which, dependent on its character, will be minimised or maximised;
$\mathbf{X}$ – array of dependent variables of the whole IPS, determined by the non-linear array equation of the whole IPS:

$$\mathbf{W}\left(\mathbf{X}, \mathbf{Y}, \mathbf{D}\right) = 0 \tag{5}$$

as an implicit function $\mathbf{X}(\mathbf{Y}, \mathbf{D})$ of the array of independent variables (controls) **Y** and of the array of initial information **D**.

Thereby, the index $\mathrm{F_s}, \forall \mathrm{s} \in \mathrm{I^R}$ is an explicit function of the array $\mathbf{Z} = (\mathbf{X}, \mathbf{Y})$ and of the array **D** in the general case. By fixed topological structure of the electrical connections between nodes of the set S_0 and fixed state of the operating equipments, array **Z** and its sub-array **Y** in particular, determine the state of IPS, i.e. not only the electrical state (values of electrical parameters), but the state of operating equipments and topological structure of electrical connections as well.

Main methodical principles of the analysis and optimisation of the control of operation and development of IPS at the interstate level by complex approach

The first main principle – application of five main clauses of GMRG [5]. At solution of the one-objective sub-problems of complex optimisation with minimisation of some function $F(\mathbf{Y})$, and at continuous change of variables, the main roles are played by the *first main clause* of GMRG about application of the theory of implicit functions, i.e. by the second form of the mathematical description of those sub-problems in particular, and even above the sub-problems of complex Pareto-optimal correction of ESS IPS (1) – (4), and by the *second clause* about the choice of the initial basis [the content of the array of the dependent variables **X** in the array equation (5)] and its rational correction in the multi-step computational process for maximal acceleration of this process with provision of limitations in the form of inequalities being parallelepiped limitations (4).

In that case the *third main clause* of GMRG – forming of the array of optimal admissible descent $\mathbf{V}^{opt}(\mathbf{Y})$ – considers the existing parallelepiped limitations only. Thereby, $\mathbf{V}^{opt}(\mathbf{Y})$ will be determined in a maximal simple way as a sub-array of the anti-gradient of the objective function

$$\frac{\partial F(\mathbf{Y})}{\partial \mathbf{Y}}, \text{ i.e. } \mathbf{V}^{\mathrm{opt}}(\mathbf{Y}) \subset -\frac{\partial F(\mathbf{Y})}{\partial \mathbf{Y}}.$$

The fourth main clause of GMRG concerns the formation of the array $\mathbf{V}^{opt}(\mathbf{Y})$ in the degeneracy situation, in which a part of the essential limitations in the form of inequalities must be determined straight by dependent variables, and its *fifth clause* on minimisation of the objective function in the direction of the array $\mathbf{V}^{opt}(\mathbf{Y})$, considering the limitation in the form of inequalities.

The second main principle at discrete optimisation, determining the number of units operating in parallel, for instance, is application of GMRG with continuous idealisation of discretely changing variables [6]. Thereby, for maximal acceleration of the computing process, a certain rational process of deformation of the ESS is applied, concerning the application of continuous idealisation of discretely variable parameters and the methods of adaptive equalizing for reducing the dimensions of the sub-problem of optimal correction of the solution in the process of that deformation [7, 8].

The third main principle – generalisation and development of mathematical tools of GMRG according to the first main principle, for different application conditions of multi-objective and game approaches and, in particular, with consideration of the following [3, 4, 9-11]:

- search for the Pareto-optimal solutions in common interests of a certain coalition of partners (EPS of Baltic States in the scope of a pool, in particular) with application of the max-min criteria (outside the Pareto-optimal control domain);
- search for the optimal compromise solutions in the common interests of a certain coalition of partners (EPS of Baltic States in the scope work of a pool, in particular) (inside the Pareto-optimal domain);
- searches for the best solutions in the conditions of different coalitions of partners (IPS *Baltija* and IPS of neighbouring states, for instance), not having sufficient unity of interests based on the max-min criteria reflecting those interests as well.

Thereby, these mathematical tools are applied for determination of the rational market mechanism, in particular for selection of optimisation criteria, boundary conditions, tariffs etc., using main approaches of the game theory, although the methods of the multi-objective and Pareto-optimal control for implementation and correction of these mechanisms, considering both normal and abnormal (crisis and emergency) situations etc. in the free market conditions, could have special importance in the future for other regions of the world.

Let us consider more precisely a complicated case of the Pareto-optimal correction of the equilibrium state of IPS as an example. Let it be:

$\mathbf{Y}_0$ – initial point (equilibrium state of IPS), such that $\mathbf{Y}_0 \notin S_{par}^{opt}$, where S_{par}^{opt} is the Pareto-optimal domain;

$\mathbf{Y}$ – some arbitrary point in the domain S_{par}^{opt}, i.e. $\mathbf{Y} \in S_{par}^{opt}$. In this case

$$D_s(\mathbf{Y}) = \left| F_s(\mathbf{Y}) - F_s(\mathbf{Y}_0) \right| \tag{6}$$

is called an improvement of the objective function $F_s(\mathbf{Y})$ at transition from the point $\mathbf{Y}_0$ in the $\mathbf{Y} \in S_{par}^{opt}$ (the absolute value is used therefore that in the case of $\max_{\mathbf{Y}} F_s(\mathbf{Y})\, F_s(\mathbf{Y}) - F_s(\mathbf{Y}_0)$ is positive, whereas by $\min_{\mathbf{Y}} F_s(\mathbf{Y})$ it is negative).

In this case the complex ESS IPS optimal correction sub-problem in its general form is reduced to the following:

$$\max_{\mathbf{Y} \in S_{par}^{opt} \cap S_Y} \; \min_{s \in \mathbf{I}^R} D_s(\mathbf{Y}), \tag{7}$$

where S_Y is the set of admissible points of the array Y, determined by the limitations (3) and (4).

Mathematical tools of GMRG (see above) could be generalised and developed for the immediate solution of the max-min problems (7). Thereby, the approximation of the Pareto-optimal domain S_{par}^{opt} in the scope of GMRG multi-step computation process is essential. Such path is theoretically prospective for the etalon modelling of the complex optimal multi-objective correction of IPS ESS, however, in the same time, especially at the beginning stage of the development of this tool, it is quite labour consuming.

Essentially more simple, even in the sense of the computing process, and in most cases giving a sufficient accurate solution, is the way of improving the state of *N* partners in the multi-step computing process of GMRG, not with straight solution of the max-min problem, starting from the initial equilibrium point $\mathbf{Y_0}$ and using (instead of the array of the multi-objective admissible descent or accent $\mathbf{V}^{opt}(\mathbf{Y}_q)$ in the case of one-criteria sub-problem in some point $\mathbf{Y}_q$ after making of *q* steps) the array of admissible multi-objective admissible descent or accent [respectively minimisation or maximisation of the function $F_s(\mathbf{Y})$], $\mathbf{V}_{par}^{opt}(\mathbf{Y}_q)$, for determination of which at each step the max-min principle of the improvement of the state of partners is used.

Let us consider some point $\mathbf{Y} = \mathbf{Y}_q \notin S_{par}^{opt}$ for execution of the next step $q+1$ t_{q+1} in direction of the array $\mathbf{V}_{par}(\mathbf{Y}_q, \mathbf{A})$, which has to led to the reduction or growth of all objective functions, min or max in (1) respectively while moving in the direction of the Pareto-optimal domain S_{par}^{opt}.

Obviously, in the case of maximisation, such array $\left\{ \frac{\partial F_s(\mathbf{Y}_q)}{\partial \mathbf{Y}} \mid s \in \mathbf{I}^R \right\}$ could be presented as some linear combination of the gradients

$$\left\{ \frac{\partial F_s\left(\mathbf{Y}_q\right)}{\partial \mathbf{Y}} \mid s \in \mathbf{I}^R \right\} \tag{8}$$

i.e.

$$\mathbf{V}_{par}\left(\mathbf{Y}_q,\ \mathbf{A}\right) = \left\{ \frac{\partial F_s\left(\mathbf{Y}_q\right)}{\partial \mathbf{Y}} \alpha_s \mid s \in \mathbf{I}^R \right\}, \tag{9}$$

where

$$\mathbf{A} = \left\{ \alpha_s \mid s \in \mathbf{I}^R \right\} \tag{10}$$

is an array of coefficients, for which the limitations

$$\mathbf{A} \geq 0 \ \ or \left\{ \alpha_s \geq 0 \mid s \in \mathbf{I}^R \right\} \tag{11}$$

and

$$\sum_{s \in \mathbf{I}^R} \alpha_s = 1 \tag{12}$$

are valid.

In the case of minimisation in (1), instead of gradients their anti-gradients are considered.

The limitations (11) and (12) determine the set of possible points $S_{\mathbf{V}\ par}\left(\mathbf{Y}_q\right)$ of the array $\mathbf{V}_{par}\left(\mathbf{Y}_q,\ \mathbf{A}\right)$, in which it is required to search for a point of the multi-objective array $\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_{\mathrm{q}}\right) \in \mathrm{S}_{\mathbf{V}\ \mathrm{par}}\left(\mathbf{Y}_q\right)$ for the search of the Pareto-optimum. Thereby the limitation (11) excludes even linear combinations (9), which, obviously, do not lead to the decrease or increase of all objective functions (1), dependent on their maximisation or minimisation.

In this case it is possible to prove that, according to (9), such an array of optimal accent or descent, dependent on maximisation or minimisation in the set-up (1) without consideration of limitations in the form of inequalities (3) and (4), could be expressed as follows:

$$\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right) = \mathbf{V}_{par}\left(\mathbf{Y}_q, \mathbf{A}_{par}^{opt}\right), \tag{13}$$

where $\mathbf{A} = \mathbf{A}_{par}^{opt}$ is determined as a solution of the following auxiliary optimisation sub-problem:

$$\min_{\mathbf{A}} \left\{ \left| \mathbf{V}_{par}\left(\mathbf{Y}_q,\ \mathbf{A}\right) \right| \mathbf{A} \geq 0;\ \sum_{s \in \mathbf{I}^R} \alpha_s = 1 \right\}. \tag{14}$$

Here the extension and development of the GMRG tool in transition from the single-criteria sub-problems of the IPS ESS complex optimisation to the solution of corresponding multi-objective improvement of the IPS ESS, where the array $\mathbf{Y}$ in changing from the initial state $\mathbf{Y}_0$ to the corresponding Pareto-optimal point $\mathbf{Y}_{par}^{opt}\left(\mathbf{Y}_0\right)$, is leading to the following:

1) instead of the gradient of the objective function $\frac{\partial F\left(\mathbf{Y_q}\right)}{\partial \mathbf{Y}}$ by one-criterion approach, the main role here by solution of the multi-objective approach is played by the array of the multi-objective accent or descent (not considering the limitations in the form of inequalities (3) and (4) $\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right)$. Thereby $\left|\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right)\right| \to 0$, if $\mathbf{Y}_q \to \mathbf{Y}_{par}^{opt}$;
2) it could be proved that with realization of the third main clause of GMRG, the array of the multi-objective accent or descent $\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right)$, considering the parallelepiped limitations in the form of inequalities (3), will be formed exactly in the same way as without consideration of these limitations, and, for the sub-array of independent variables of lower dimension $\mathbf{Y}' \subset \mathbf{Y}$, the contents of the components of which will be determined on the basis of the array $\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right)$ of complete dimensions;
3) it could be shown that to avoid the degeneration, requiring realisation of the third main clause of GMRG, the role of an efficient change of the basis will essentially increase, in which case the determination of the array $\mathbf{V}_{par}^{opt}\left(\mathbf{Y}_q\right)$ will be much more complicated, and the multi-step computation process will be essentially slowed down;
4) it could be shown that in realisation of the fifth main clause of GMRG, the same mathematical tool as at solution of one-objective sub-problems of complex optimisation could be used. However, from multiple possible steps on the search of the extreme of the objective functions of different partners the minimal one must be selected to avoid degradation of the objective function of any of partners.

Such multi-step process of the multi-objective improvement of IPS ESS in the scope of main principles of GMRG is called Pareto-optimal correction, i.e. correction of ESS of IPS which leads, integrated and with given accuracy, to the multi-objective improvement in accordance with the mini-max set-up of the problem (7), where the array $\mathbf{Y}_q$ approaches from the point $\mathbf{Y}_0$ to the Pareto-optimal domain S_{par}^{opt} in the point $\mathbf{Y}_{par}^{opt}$.

The fourth main principle of the multi-objective optimisation, and Pareto-optimal correction in particular, considering discrete variation of parameters, is reduced, as in the case of the second principle, to the continuous idealisation of discretely varying parameters and to the adaptive equalizing. However, the strict adjustment of approximate solutions within the scope of the abovementioned continuous idealisation in the deformation process of equations is realised with approximation of the objective, using the integral form of the least square method and its last developments – orthogonal wave functions [3, 4, 10, and 12]. Here, for acceleration of the computing process in the scope of GMRG for precision of the approximate Pareto-optimal solution, proceeding from the process of logarithmic mean (i.e. suppression of narrow waves by totalizing of certain sub-totals of orthogonal series, based on the weight logarithmic means), a certain combination with the new developed method – the method of control plane – is applied. This move enables to eliminate from the search inappropriate discrete points using large series, without the need of their thorough examination.

The fifth main principle lies in the rational Pareto-optimal correction of the ESS of partners within the scope of their separate coalitions, considering nonantagonistic conflicts between coalitions (in the case of parallel operation of IPS *Baltija* with IPS of neighbouring countries, for instance), based on the development of the general form of game theory [10, 11, 13]. This is expressed in the following:

1) in the general case – in application of income (or costs) function and certain max-min (min-max) criteria for rational correction of Pareto-optimal solutions [this is directed towards maximisation of the total income (or minimisation of the total costs) for different coalitions of partners], taking into account the possibilities of unsuitable changes of the equations on the boundaries of neighbouring coalitions in certain sub-regions, which could be iteratively adjusted in the development process of the cooperation;
2) in elaboration of a corresponding method of composition of the iterative game process;
3) in elaboration of the solution methods for corresponding max-min (or min-max) sub-problems, which are based on GMRG and on the development of its mathematical tools for determination of solving functions for acceleration of the correction of Pareto-optimal solutions.

The next main problem lies in the evaluation method of the sub-domains of possible changes of equations on the borders with neighbouring coalitions, taking into account the iterative development process of the cooperation of IPS coalitions.

In the studies on the market games the max-min principle is generally used. According to this principle, every gambler (or a coalition as well) considers each of its strategies, such a strategy of his counter players, which could harm him most in the given situation, and, proceeding from this, elaborates his most favourable strategy. In the given case, it is an antagonistic game with collisions between gamblers (or coalitions). Although some of investigators do mention that not all other gamblers wish always to cause maximum loss for us, but just want to get the maximum profit, which is not reflected in the zero-sun games. Nevertheless, a corresponding research of the market relations is not known until now.

Thereby, the research carried out by BASRE has made the first step on correction of max-min principle in the direction leading to the iterative adjustment of the strategy of gamblers, basing on the modification of the characteristic function, and it could be considered a new methodological approach to elaboration of a corresponding generalized method at the level of IPS coalitions [10, 13].

The sixth main principle lies in application of the optimality principle in the reduction of multi-stage hierarchical problems of dynamic programming to solution of separate sub-problems of complex optimisation on the basis of principles 1–5 [3, 4]. Thereby, the optimal control of ESS in its general form is related to consideration of the multi-stage hierarchical sub-problems, which differ in the time, space, and situation aspects. A similar approach is also used in optimisation of the development of IPS. Alternatively, on the other side, this approach enables to consider the application of main principles 1–5 for solution of multi-stage hierarchical problems of dynamic programming.

Conclusion

The main principles and methods represent the kernel in development of the theory and methods of complex optimisation of the control and of operation reliability of interconnected power systems (IPS) at interstate level in free market conditions, considered by the authors in [1].

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