Proc. Estonian Acad. Sci. Phys. Math., 1993, 42, 1, 22—31
https://doi.org/10.3176/phys.math.1993.1.03

UDC 59.3

Helle HEIN and Jaan LELLEP**

OPTIMAL PLASTIC DESIGN OF SHALLOW SHELLS WITH STEP-WISE VARYING CROSS SECTION

(Presented by J. Engelbrecht)

Abstract. The problem of minimum weight design of thin shallow spherical shells with piece-wise constant thickness is studied. The material of the shell is rigid, perfectly plastic, obeying Tresca yield condition and the associated deformation law. The influence of geometry changes which occur in the post-yield stage, is taken into account. The optimization problem is posed under the condition that the deflections of the optimized shell and of a reference shell of constant thickness, respectively, coincide. Necessary optimality conditions are derived with the aid of the variational methods of the optimal control theory.

1. Introduction

Optimal design of rigid-plastic structures has usually been studied in the case of collapse load — thus, under the requirement of incipient plastic flow. Such designs appear to be sensitive to geometrical changes taking place in the post-yield range [1].

Introducing the geometrical nonlinearity one can examine the post-yield behaviour of the structure. In the present work it is assumed that the strains of the thin shells remain infinitesimal whereas the transverse deflections being finite do not exceed the order of the shell wall thickness.

Large deformations of plastic structures have been investigated by several authors [2]. The post-yield stage of shallow shells of Tresca material was studied by Duszek [3]. The same problem was solved by Sherbourne and Haydl [4] in the case of von Mises yield condition. Kondo and Pian [5] suggested a method on the basis of the assumption that the shell deforms into a number of truncated cones which are separated by plastic hinge circles. The authors [6] used an approximation of the exact yield surface which consists in the separation of the bending and membrane responses.

Different optimization techniques for geometrically non-linear axi-symmetric plates and shells have been suggested in [2, 7, 8].

In the present study the optimal design problem is examined by an approximate method employed earlier by the authors [7] for investigating circular and annular plates of piece-wise constant thickness. The optimal thickness variation is sought for a shell under the requirement of minimum material consumption.

2. Formulation of the problem

Let us consider a simply supported thin shallow spherical shell of radius A (Fig. 1). It is assumed that the structure is axisymmetric and

$$h=h_j \tag{2.1}$$

* Tartu Ülikool (Tartu University). Vanemuise 46, EE-2400 Tartu. Estonia.

for $r \in D_j$ $(j=0, \dots, n)$. Here D_j stands for an interval (a_j, a_{j+1}) and $a_0=0$, $a_{n+1}=R$. Let the shell be subjected to the uniformly distributed external pressure loading of intensity P. The geometrical parameters and the loading will be considered as the given constants.

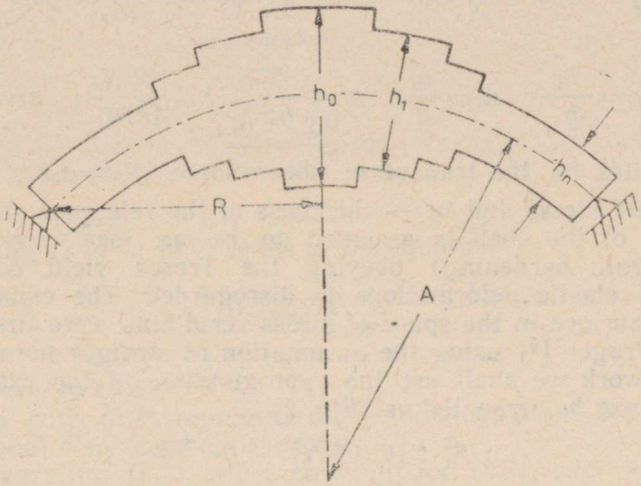

Fig. 1. Shell geometry.

We are looking for a design of the shallow spherical shell for which the material volume

$$V=\sum_{i=0}^{n} h_i \left(\sqrt{A^2-a_i^2}-\sqrt{A^2-a_{i+1}^2}\right) \tag{2.2}$$

attains the minimal value under the condition that the deflections of the shells of piece-wise constant thickness and of constant thickness, respectively, coincide.

The stress-strain state of the shell in the post-yield range is determined by the membrane stresses N_r, N_θ, bending moments M_r, M_θ, and radial and transverse displacements U, W.

Assuming that the displacements do not exceed the order of the shell thickness the theory of von Karman is applicable. Thus, the equilibrium equations have the form

$$\begin{aligned} &(\varrho n_1)'=n_2, \\ &[(\varrho m_1)'-m_2]'+[4\varrho n_1(\varrho t+w')]'+p\varrho=0, \end{aligned} \tag{2.3}$$

whereas the deformation components coupled with the stress components may be written as

$$\begin{aligned} &\varepsilon_1=\frac{4M_*}{tAN_*}\left(u'+t\varrho w'+\frac{1}{2}w'^2\right), \\ &\varepsilon_2=\frac{4M_*}{tAN_*}\frac{u}{\varrho}, \\ &\varkappa_1=-\frac{4M_*}{R^2N_*}w'', \\ &\varkappa_2=-\frac{4M_*}{R^2N_*}\frac{w'}{\varrho}. \end{aligned} \tag{2.4}$$

In (2.3) and (2.4) the primes denote the differentiation with respect to the nondimensional coordinate ϱ. Dimensional and nondimensional quantities are coupled by the relations

$$\varrho=\frac{r}{R},\quad n_{1,2}=\frac{N_{r,\theta}}{N_*},\quad m_{1,2}=\frac{M_{r,\theta}}{M_*},\quad p=\frac{PR^2}{M_*},\quad s=\frac{A}{R},$$

$$u=\frac{UR}{h_*^2},\quad \gamma_j=\frac{h_j}{h_*},\quad a_j=\frac{a_j}{R},\quad w=\frac{W}{h_*},\quad t=\frac{R^2N_*}{4AM_*},\quad q=\frac{QR}{M_*}. \tag{2.5}$$

Here Q stands for the transverse shear force, $N_*=\sigma_0 h_*$, $M_*=\sigma_0 h_*^2/4$, σ_0 being yield stress and h_* — thickness of the reference shell.

Material of the shell is assumed to be an ideal rigid-plastic one (without strain hardening) obeying the Tresca yield condition. The influence of elastic deformations is disregarded. The equations of the exact yield surface in the space of stress resultants were first derived by Onat and Prager [9] using the assumption of straight normals.

In this work we shall use the approximation of the exact-yield surface which may be presented as (Fig. 2)

$$n_i^2+|m_i|\leqslant\gamma_j^2,\qquad i=1,2 \tag{2.6}$$

for the region D_j, $j=0, \ldots, n$. On the m_1-n_1 plane we shall use the generalized square yield condition which may be conceived as a linear approximation to the current yield surface.

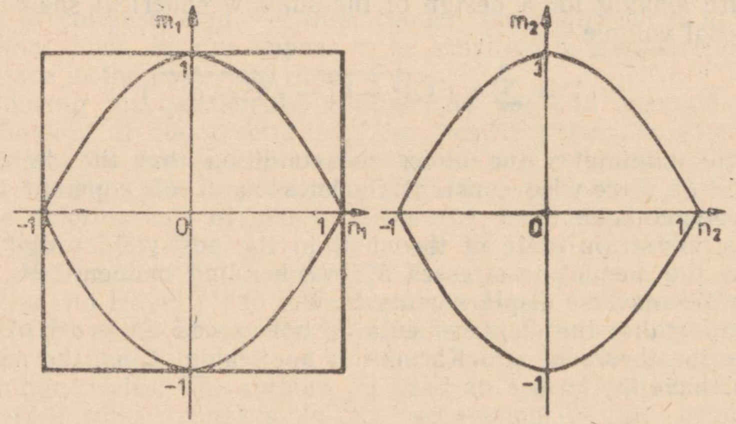

Fig. 2. Yield condition.

A deformation-type theory of plasticity will be utilized in the present study. The relations between generalized stresses and strain components are furnished by the associated deformation law which states that the vector with strain components (2.4) is directed along the outward normal to the yield curve (2.6).

Since the outer edge of the shell is hinged, the boundary conditions may be expressed as

$$m_1(1)=u(1)=w(1)=q(0)=0,$$
$$w(0)=w_0. \tag{2.7}$$

At the centre of the shell the symmetry conditions will take the following form

$$m_1(0)=m_2(0),\qquad n_1(0)=n_2(0). \tag{2.8}$$

3. Necessary optimality conditions

Let us assume that the plastic behaviour of the shell corresponds to the following flow regime: $|n_1| \leqslant \gamma_j$, $|m_1| \leqslant \gamma_j^2$, $n_2^2 + |m_2| = \gamma_j^2$ for each $\varrho \in D_j$. According to the associated deformation law,

$$\varepsilon_1 = \varkappa_1 = 0 \tag{3.1}$$

and

$$\frac{\varepsilon_2}{\varkappa_2} = \frac{2M_*}{N_*} n_2 \tag{3.2}$$

for each $\varrho \in D_j$ $(j = 0, \ldots, n)$.

Substituting (2.4) in (3.2) leads to the equation

$$n_2 = -2 \frac{u}{w'} \tag{3.3}$$

which holds everywhere.

Eqs. (3.1) with (2.4) represent differential equations with respect to non-dimensional displacements u and w.

When integrating (3.1) and accounting for (2.7), one can state that

$$w = w_0 (1 - \varrho) \tag{3.4}$$

and

$$u = \frac{w_0}{2} (\varrho - 1)(t(\varrho + 1) - w_0) \tag{3.5}$$

for $\varrho \in (0, 1)$. Combining (3.4), (3.5) and (3.3) gives

$$n_2 = w_0 (1 - \varrho) + t(\varrho^2 - 1). \tag{3.6}$$

The circumferential bending moment m_2 can be expressed with the aid of (2.6) and (3.6) in the form

$$m_2 = \gamma_j^2 - \{w_0 (1 - \varrho) + t(\varrho^2 - 1)\}^2 \tag{3.7}$$

since the bending moment is assumed to be non-negative.

Substituting (3.6) into the first equation in (2.3), one obtains

$$n_1' = -\frac{n_1}{\varrho} + w_0 \left(\frac{1}{\varrho} - 1 \right) + t \left(\varrho - \frac{1}{\varrho} \right) \tag{3.8}$$

for $\varrho \in (0, 1)$. The second equation in the set (2.3) can be integrated making use of (2.7), and the continuity requirements imposed on q, n_1, w at $\varrho = \alpha_i$ give

$$(\varrho m_1)' - m_2 + 4\varrho n_1 (t\varrho + w') + \frac{p}{2} \varrho^2 = 0. \tag{3.9}$$

Inserting (3.7) and taking (3.4) into account, the Eq. (3.9) can be expressed as

$$m_1' = -\frac{m_1 - \gamma_i^2}{\varrho} - \varrho \left\{ w_0 \left(\frac{1}{\varrho} - 1 \right) + t \left(\varrho - \frac{1}{\varrho} \right) \right\}^2 -$$

$$- 4n_1 (t\varrho - w_0) - \frac{p}{2} \varrho. \tag{3.10}$$

Making use of (2.4), (3.4), (3.5) it is easy to recheck that $\varkappa_2 \geqslant 0$ and $u \leqslant 0$ if $w_0 \leqslant t$, e. g. the stress profile appears to be kinematically admissible.

For statical admissibility of the stress state, it is necessary that

$$\begin{aligned} |n_1| &\leqslant \gamma_i, \quad \varrho \in D_i, \\ 0 \leqslant m_1 &\leqslant \gamma_i^2, \quad \varrho \in D_i. \end{aligned} \tag{3.11}$$

The most dangerous places where (3.11) might be violated appear to be the circles associated with the steps in the thickness. Assuming that the membrane stress n_1 is non-positive, the constraints (3.11) can be substituted with

$$\begin{aligned} m_1(\alpha_i) &\leqslant \gamma_i^2, \\ n_1(\alpha_i) &\geqslant -\gamma_i, \quad \varrho \in D_i \end{aligned} \tag{3.12}$$

or

$$\begin{aligned} n_1(\alpha_i) + \gamma_i - \xi_i^2 &= 0, \\ m_1(\alpha_i) - \gamma_i^2 + \theta_i^2 &= 0, \end{aligned} \tag{3.13}$$

where ξ_i, θ_i stand for unknown parameters.

The problem posed above will be considered as a variational problem of the optimal control theory. Evidently, m_1 and n_1 in (3.8) and (3.10) must be referred to as the state variables, and α_j, γ_i, θ_j, ξ_j ($j=1, \ldots, n$; $i=0, \ldots, n$) as the preliminary unknown constant parameters. The quantity w_0 will be handled as the given maximum deflection of the shell with constant thickness.

The cost function (2.2) may be presented as

$$J = \sum_{i=0}^{n} \gamma_i \left(\sqrt{s^2 - \alpha_i^2} - \sqrt{s^2 - \alpha_{i+1}^2}\right). \tag{3.14}$$

When minimizing (3.14) one has to take into account the differential Eqs. (3.8), (3.10), the additional requirements (3.13) and the boundary conditions (2.7), (2.8). In order to derive necessary optimality conditions the following functional will be employed:

$$\begin{aligned} J_* = J + \sum_{i=0}^{n} \int_{D_i} (\lambda n_1' + \psi m_1' - L_i)\, d\varrho + \\ + \sum_{j=1}^{n} \{\eta_j (m_1(\alpha_j) - \gamma_j^2 + \theta_j^2) + \nu_j (n_1(\alpha_j) + \gamma_j - \xi_j^2)\}. \end{aligned} \tag{3.15}$$

Here λ and ψ will be considered as the adjoint variables and η_j, ν_j stand for the unknown constant Lagrangian multipliers. The functions λ, ψ will be regarded as piecewise continuous functions.

In (3.15) due to (3.8) and (3.10) the Lagrangian function is expressed as

$$\begin{aligned} L_i = \lambda \left\{ -\frac{n_1}{\varrho} + w_0\left(\frac{1}{\varrho} - 1\right) + t\left(\varrho - \frac{1}{\varrho}\right)\right\} + \psi \left\{ -\frac{m_1 - \gamma_i^2}{\varrho} - \right. \\ \left. - \varrho \left\{ w_0\left(\frac{1}{\varrho} - 1\right) + t\left(\varrho - \frac{1}{\varrho}\right)\right\}^2 - 4n_1(t\varrho - w_0) - \frac{p}{2}\varrho \right\}. \end{aligned} \tag{3.16}$$

For optimality of the solution it is necessary that the total variation of the functional (3.16) should be equal to zero. The variations of the state variables at the points α_j will be determined by the following expressions

$$\Delta m_1(\alpha_j\pm)=\delta m_1(\alpha_j\pm)+\frac{dm_1}{d\varrho}\bigg|_{\rho=\pm\alpha_j}\cdot\Delta\alpha_j,$$
$$\Delta n_1(\alpha_j\pm)=\delta n_1(\alpha_j\pm)+\frac{dn_1}{d\varrho}\bigg|_{\rho=\pm\alpha_j}\cdot\Delta\alpha_j, \qquad (3.17)$$

where δm_1, δn_1 stand for the weak variations and Δm_1, Δn_1 denote the total variations of the functions m_1, n_1; $\Delta\alpha_j$ being the increments of the parameters α_j. It is assumed that the quantities m_1 and n_1 are continuous from which yields

$$\Delta m_1(\alpha_j+)=\Delta m_1(\alpha_j-)=\Delta m_1(\alpha_j),$$
$$\Delta n_1(\alpha_j+)=\Delta n_1(\alpha_j-)=\Delta n_1(\alpha_j). \qquad (3.18)$$

However the left and right-hand weak variations at $\varrho=\alpha_j$ $\delta n_1(\alpha_j\pm)$, $\delta m_1(\alpha_j\pm)$ must not be equal to each other. Moreover, the adjoint coordinates λ, ψ may have finite jumps at the points α_j $(j=1, \ldots, n)$.

Performing the differentiation of the functional (3.15), one obtains

$$\sum_{j=1}^{n}\frac{\partial J}{\partial\alpha_j}\Delta\alpha_j+\sum_{j=0}^{n}\frac{\partial J}{\partial\gamma_j}\Delta\gamma_j+\sum_{i=0}^{n}\int_{D_i}\Big\{-\lambda'\delta n_1-\psi'\delta m_1-\frac{\partial L_i}{\partial n_1}\delta n_1-$$
$$-\frac{\partial L_i}{\partial m_1}\delta m_1-\sum_{j=0}^{n}\frac{\partial L_i}{\partial\gamma_j}\Delta\gamma_j\Big\}d\varrho+\sum_{j=0}^{n}(\lambda\delta n_1+\psi\delta m_1)\Big|_{\alpha_i}^{\alpha_{j+1}}+$$
$$+\sum_{j=1}^{n}\{\eta_j(\Delta m_1(\alpha_j)-2\gamma_j\Delta\gamma_j+2\theta_j\Delta\theta_j)+\nu_j(\Delta n_1(\alpha_j)+\Delta\gamma_j-2\xi_j\Delta\xi_j)\}=0. \qquad (3.19)$$

Bearing in mind that δn_1 and δm_1 are the independent variations of corresponding quantities in (3.19), the adjoint set could be presented as

$$\lambda'=\frac{\lambda}{\varrho}+4t\psi\varrho,$$
$$\psi'=\frac{\psi}{\varrho} \qquad (3.20)$$

for each subdomain D_j. According to (2.7), (2.8) and (3.19) the transversality condition takes the form

$$\lambda(1)=0. \qquad (3.21)$$

Since $\Delta\gamma_j$ are arbitrary variations, from (3.19) and (3.4) follow the equations

$$\sqrt{s^2-\alpha_j^2}-\sqrt{s^2-\alpha_{j+1}^2}-\int_{D_j}2\gamma_j\frac{\psi}{\varrho}d\varrho-2\gamma_j\eta_j+\nu_j=0. \qquad (3.22)$$

Due to the independence of variations $\Delta\theta_j, \Delta\xi_j$ one has

$$\eta_j\theta_j=0,$$
$$\nu_j\xi_j=0. \qquad (3.23)$$

Substituting (3.20)—(3.23) into (3.19) and taking into account (3.14), (3.17), (3.18) leads to

$$\frac{\alpha_j(\gamma_{j-1}-\gamma_j)}{\sqrt{s^2-\alpha_j^2}}\Delta\alpha_j+\sum_{i=0}^{n}\Big\{\lambda(\alpha_{i+1}-)(\Delta n_1(\alpha_{i+1})-n_1'(\alpha_{i+1}-)\Delta\alpha_{i+1})-$$
$$-\lambda(\alpha_i+)(\Delta n_1(\alpha_i)-n_1'(\alpha_i+)\Delta\alpha_i)+\psi(\alpha_{i+1}^-)(\Delta m_1(\alpha_{i+1})-$$
$$-m_1'(\alpha_{i+1}-)\Delta\alpha_{i+1})-\psi(\alpha_i+)(\Delta m_1(\alpha_i)-m_1'(\alpha_i+)\Delta\alpha_i)\Big\}+$$
$$+\sum_{i=1}^{n}\Big\{\eta_i\Delta m_1(\alpha_i)+\nu_i\Delta n_1(\alpha_i)\Big\}=0, \tag{3.24}$$

which combined with (3.15) and (3.17) yields

$$\frac{\alpha_j(\gamma_{j-1}-\gamma_j)}{\sqrt{s^2-\alpha_j^2}}\Delta\alpha_j-\sum_{i=1}^{n}\Big\{\Delta n_1(\alpha_i)[\lambda(\alpha_i)]+\Delta m_1(\alpha_i)[\psi(\alpha_i)]-$$
$$-L_i(\alpha_i)\Delta\alpha_i+L_{i-1}(\alpha_i)\Delta\alpha_i-\eta_i\Delta m_1(\alpha_i)-\nu_i\Delta n_1(\alpha_i)\Big\}=0. \tag{3.25}$$

The square brackets in (3.25) stand for the finite discontinuities of corresponding variables:

$$[\lambda(\alpha_i)]=\lambda(\alpha_i+)-\lambda(\alpha_i-),$$
$$[\psi(\alpha_i)]=\psi(\alpha_i+)-\psi(\alpha_i-). \tag{3.26}$$

Considering $\Delta m_1(\alpha_j)$, $\Delta n_1(\alpha_j)$ for $j=1, \ldots, n$ as arbitrary variations in (3.24), one obtains

$$[\lambda(\alpha_j)]=\nu_j \tag{3.27}$$

and

$$[\psi(\alpha_j)]=\eta_j. \tag{3.28}$$

Thus the adjoint variables have finite jumps at the boundary points of regions D_j.

Similarly, due to the arbitrariness of the increments $\Delta\alpha_j$ in (3.25), it follows that

$$\frac{\alpha_j(\gamma_{j-1}-\gamma_j)}{\sqrt{s^2-\alpha_j^2}}+L_j(\alpha_j)-L_{j-1}(\alpha_j)=0 \tag{3.29}$$

for $j=1, \ldots, n$.

4. Optimal design of a shallow spherical shell with hinged outer edge

Necessary optimality conditions for the posed problem are presented by (3.20)—(3.22), (3.27)—(3.29). The optimal design of the shell is such that the equilibrium Eqs. (2.3) and boundary conditions (2.7), (2.8) are met as well.

According to (3.23) we have

$$\eta_j=0, \quad \theta_j\neq 0 \tag{4.1}$$

or

$$\eta_j\neq 0, \quad \theta_j=0 \tag{4.2}$$

and

$$\nu_j=0, \quad \zeta_j\neq 0 \tag{4.3}$$

or

$$\nu_j\neq 0, \quad \zeta_j=0. \tag{4.4}$$

Let us assume that the optimal design corresponds to (4.2) and (4.3). It means that the adjoint variable λ is continuous everywhere.

Performing the integration in (3.20) one obtains

$$\begin{aligned} \psi &= \psi_i \varrho, \\ \lambda &= (2t\psi_i \varrho^2 + E_i)\varrho \end{aligned} \tag{4.5}$$

for $\varrho \in D_i$, $i=0, \ldots, n$. Taking into account (3.21) and the continuity requirements, one can determine the integration constants as

$$\begin{aligned} &E_{i-1} = E_i + 2t\alpha_i^2(\psi_i - \psi_{i-1}), \qquad i=n, n-1, \ldots, 1, \\ &E_n = -2t\psi_n. \end{aligned} \tag{4.6}$$

Substituting (4.5) into (3.22) and performing the integration leads to the following equations

$$\sqrt{s^2-\alpha_j^2} - \sqrt{s^2-\alpha_{j+1}^2} + 2\gamma_j(\psi_{j-1}\alpha_j - \psi_j\alpha_{j+1}) = 0, \tag{4.7}$$

where $j=0, \ldots, n$. Making use of (3.6) and (2.8), one can integrate the Eq. (3.8) to give

$$n_1 = w_0\left(1 - \frac{\varrho}{2}\right) + t\left(\frac{\varrho^2}{3} - 1\right). \tag{4.8}$$

The second equation in (2.3), after substitutions of the quantities m_2, n_1, w' according to (3.7), (4.8), (3.4), gives

$$\begin{aligned} m_1 = &-\frac{7}{15}t^2\varrho^4 + \frac{4}{3}w_0 t\varrho^3 - \left(w_0^2 + 2w_0 t - 2t^2 + \frac{p}{6}\right)\varrho^2 + \\ &+ 3w_0(w_0 - t)\varrho + \gamma_j^2 - (w_0 - t)^2 + B_j + \frac{C_j}{\varrho}, \qquad \varrho \in D_j. \end{aligned} \tag{4.9}$$

The integration constants B_j, C_j can be determined with the aid of the continuity requirements of the bending moment m_1 and the shear force at $\varrho = \alpha_j$ and (2.8). Therefore,

$$\begin{aligned} &B_j = C_0 = 0, \\ &C_j = C_{j-1} - \alpha_j(\gamma_j^2 - \gamma_{j-1}^2), \qquad j=1, \ldots, n. \end{aligned} \tag{4.10}$$

Combining (4.9), (4.2), (3.13) gives

$$\begin{aligned} C_j &- \frac{7}{15}t^2\alpha_j^5 + \frac{4}{3}w_0 t\alpha_j^4 - \left(w_0^2 + 2w_0 t - 2t^2 + \frac{p}{6}\right)\alpha_j^3 + \\ &+ 3w_0(w_0 - t)\alpha_j^2 - (w_0 - t)^2\alpha_j = 0, \qquad j=1, \ldots, n. \end{aligned} \tag{4.11}$$

Making use of (3.16) and (4.5), the equation (3.29) may be presented as

$$\begin{aligned} &\frac{\alpha_j(\gamma_{j-1} - \gamma_j)}{\sqrt{s^2 - \alpha_j^2}} - \psi_j\alpha_j\left[\frac{m_1(\alpha_j) - \gamma_j^2}{\alpha_j} + \alpha_j\left(w_0\left(\frac{1}{\alpha_j} - 1\right) + t\left(\alpha_j - \frac{1}{\alpha_j}\right)\right)^2 + \right. \\ &\left. + 4n_1(\alpha_j)(t\alpha_j - w_0) + \frac{p}{2}\alpha_j\right] + \psi_{j-1}\alpha_j\left[\frac{m_1(\alpha_j) - \gamma_{j-1}^2}{\alpha_j} + \right. \\ &\left. + \alpha_j\left(w_0\left(\frac{1}{\alpha_j} - 1\right) + t\left(\alpha_j - \frac{1}{\alpha_j}\right)\right)^2 + 4n_1(\alpha_j)(t\alpha_j - w_0) + \frac{p}{2}\alpha_j\right] = 0. \end{aligned} \tag{4.12}$$

Finally, the boundary condition for the bending moment m_1 in (2.7) yields

$$C_n+\frac{8}{15}t^2-\frac{5}{3}w_0t+w_0^2-\frac{p}{6}+\gamma_n^2=0. \qquad (4.13)$$

The set of equations (4.6), (4.7), (4.10)—(4.13) serves for determining the unknown constants ψ_i, E_i, C_i and parameters α_j, γ_i ($i=0, \ldots, n$; $j=1, \ldots, n$).

5. Discussion

The numerical results for the shell with two thicknesses are presented in the Table. The geometrical parameters have the following values: $t=0.7$; $s=2$.

Optimal parameters of the shell with one step in the thickness

$\|W_0\|$	$\|p\|$	α_1	γ_0	γ_1	e
0	9.200	0.696	1.098	0.725	0.9054
0.1	10.460	0.738	1.083	0.716	0.9095
0.2	11.840	0.772	1.071	0.708	0.9235
0.3	13.340	0.799	1.062	0.701	0.9307
0.4	14.960	0.822	1.0545	0.695	0.9370

The economy of the established optimal design could be assessed by the coefficient

$$e=\frac{J}{V_*},$$

where J is the optimal volume (3.14) and V_* stands for the volume of the reference shell with constant thickness:

$$V_*=s-\sqrt{s^2-1}.$$

The values of the economy coefficient are accommodated in the last column of the Table. The calculations carried out show that the utilization of the structure with two different thicknesses enables to save 9.5% of the material, provided the shell operates in the limit state. The saving decreases with the increase of deflections.

REFERENCES

1. *Mroz, Z. and Gawecki, A.* In: Optimization in Structural Design (Ed. A. Sawczuk and Z. Mroz). Springer, Heidelberg et al., 1975, 518—540.
2. *Lellep, J.* Optimization of Plastic Structures. Tartu Univ. Press, Tartu, 1991.
3. *Duszek, M.* In: Theory of Thin Shells (Ed. F. Niordson). Springer, Heidelberg et al., 1969, 374—388.
4. *Sherbourne, A. N. and Haydl, H. M.* ZAMM, 1974, **54**, 2, 73—82.
5. *Kondo, K. and Pian, T. H. H.* Int. J. Mech. Sci., 1981, 22, 69—76.
6. *Lellep, J. and Hein, H.* Tartu Riikl. Ülik. Toim., 1988, 799, 37—51.
7. *Lellep, J. and Hein, H.* Tartu Ülik. Toim., 1992, 939, 54—69.
8. *Lellep, J. and Sawczuk, A.* Int. J. Solids and Struct., 1987, **23**, 5, 651—664.
9. *Onat, E. T. and Prager, W.* Proc. Netherlands Acad. Sci., 1954, **B57**, 334—348.

Received
July 9, 1992

Helle HEIN, Jaan LELLEP

ASTMELISELT MUUTUVA RISTLÕIKEGA LAMEDATE PLASTSETE KOORIKUTE OPTIMAALNE PROJEKTEERIMINE

On vaadeldud astmeliselt muutuva paksusega lamedat sfäärilist koorikut, millele mõjub ühtlane välisrõhk ning mis on serva mööda šarniirselt kinnitatud. Kooriku materjal on jäikplastne ja allub Tresca tingimusele ning vastavale gradientaalsuse seadusele. Kasutades optimaalse juhtimise teooria variatsioonmeetodeid on tuletatud võrrandid selliste paksuste ja paksuse hüppekohtade määramiseks, mille korral kooriku ruumala on minimaalne etteantud koormuse ja läbipainde puhul. Saadud võrrandid on lahendatud numbriliselt.

Хелле ХЕЙН, Яан ЛЕЛЛЕП

ОПТИМАЛЬНОЕ ПРОЕКТИРОВАНИЕ ПЛАСТИЧЕСКИХ ПОЛОГИХ ОБОЛОЧЕК СО СТУПЕНЧАТЫМ ПОПЕРЕЧНЫМ СЕЧЕНИЕМ

Рассматривается задача определения проекта минимального веса пологих сферических оболочек ступенчато-постоянной толщины, подверженных действию равномерного внешнего давления. Края оболочек считаются шарнирно закрепленными. Материал оболочек жесткопластический, подчиняющийся условию пластичности Треска и ассоциированному закону деформирования. С помощью вариационных методов теории оптимального управления выводятся условия для определения различных толщин и координат ступеней, при которых объем материала достигает минимума. Полученная система уравнений решается численно.