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MODAL RESONANCES IN PROBLEMS OF ACOUSTIC WAVES SCATTERING BY ELASTIC SHELLS

(Presented by J. Engelbrecht)

Abstract. The problem of an obliquely incident plane acoustic wave scattering by a circular cylindrical shell is considered. A procedure to isolate the modal resonances is presented for this example. An approach is outlined for the determination of the numerical characteristics of each resonance.

Introduction

As it is known [1], the resonance frequencies of the peripheral waves, generated by an incident wave in an elastic scatterer, coincide with those of partial modes. The resonance scattering theory [2–5] is an effective tool for the description of the resonance components of partial modes. According to this theory, specially chosen backgrounds are used for separating the resonance components. Firstly, the rigid and soft backgrounds have been used, and then various kinds of intermediate backgrounds have been proposed [6–10]. Formally speaking, when the type of the background is chosen, one can compute the resonance components of partial modes. Actually, the things are not so simple. Below it will be shown how one should practically act in order to compute the resonance components of partial modes and to determine the parameters describing the resonance. In this connection three questions are discussed here: the choice of the background, the choice of the computational step size, and the discrimination of closely located resonance components. All the notations used are the same as in paper [9].

1. The background specification

According to the procedure of the resonance scattering theory [2], the difference is formed between the partial form function and its background in order to separate the resonance components of a partial mode. As it is noted in [2], even with the fixed physical parameters of the scatterer (ϱ_l, c_l, c_t, and h) one cannot use one background only for all the types of peripheral waves generated in the scatterer by the incident wave. Even more, the type of the background for each peripheral wave depends generally speaking, on the frequency and the ordinal number n of the resonance.

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Usually, the computation of the resonance components of partial modes is organized in the following way. Firstly, according to the standard recommendations [2, 6] and some unrigorous heuristic considerations, the type of the background is specified for some frequency range $x_a \leqslant x \leqslant x_b$. Then the modulus of the difference $|f_n(x) - \bar{f}_n(x)|$ is calculated with some chosen computational step size l_x for successively increasing n values. Such a function has typical extrema which are numerated in succession with increasing variable x. Each extremum is labeled by two indices x_{nm} (the first index shows the ordinal number of the resonance, i. e. the number of the wave lengths which fit the path of the peripheral wave, and the second index defines the type of the wave). With the correctly specified background and sufficiently small computational step size, the coordinate x_{nm} defines the resonance frequency. Here the maximal amplitude of the partial mode is located precisely at the resonance frequency (see Fig. 1).

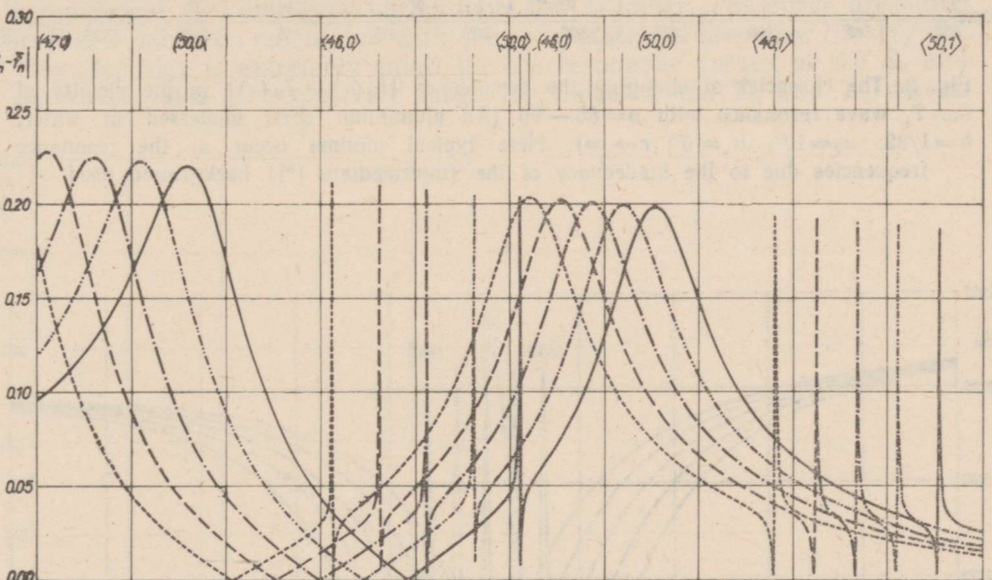

Fig. 1. Modal resonances in a bounded frequency range. The computation has been carried out for the problem of scattering of an obliquely incident plane acoustic wave by a circular cylindrical shell (An aluminium shell immersed in water; the physical parameters are defined in [10]; the relative thickness of the shell is $h = 1/10$; the angle of incidence is $\alpha_0 = 13°$, the observation angle $\alpha_* = 0°$; the observation point is situated in the far field ($r \to \infty$); the rigid background has been used here).

With successively increasing n, due to defocusing of the functions $f_{nm}(x)$ and $\bar{f}_{nm}(x)$, a situation can occur when, for the specified type of the background, a minimum will be situated on the resonance frequency instead of a maximum (see Fig. 2). It is clear that the background has become inadequate and should be changed. The resonance component can be isolated when the correct background is used (see Fig. 3).

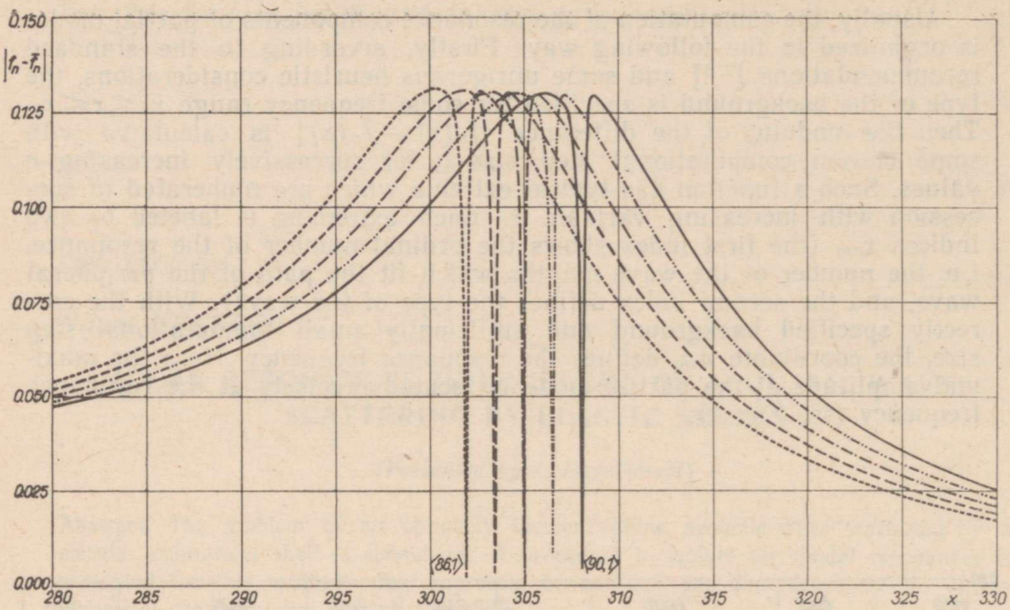

Fig. 2. The character of changing the dependence $|f_n(x) - \bar{f}_n(x)|$ in the vicinity of the T_1 wave resonance with $n=86-90$ (An aluminium shell immersed in water; $h=1/32$; $\alpha_0=10°$; $\alpha_*=0°$; $r\to\infty$). Here typical minima occur at the resonance frequencies due to the inadequacy of the (intermediate [9]) background used.

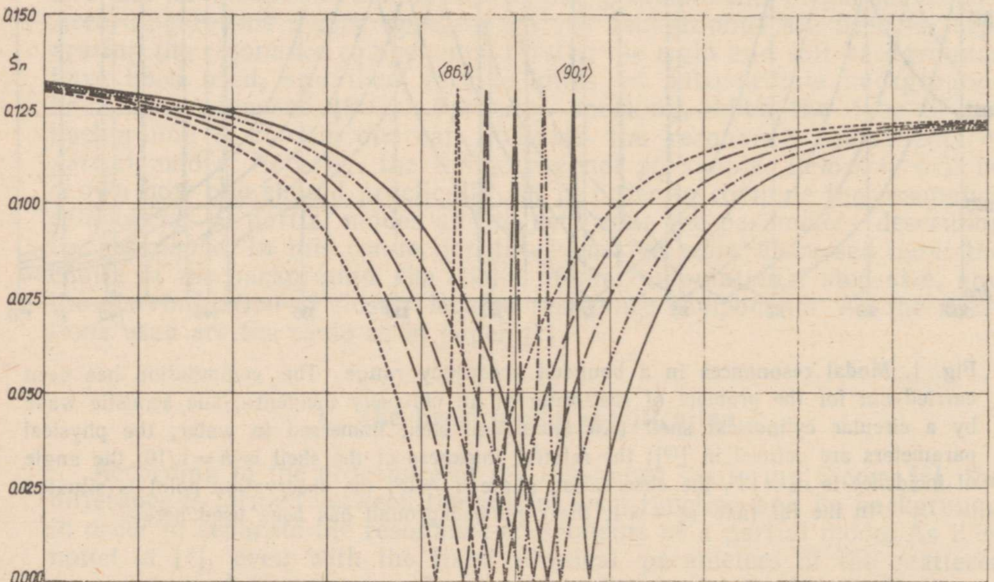

Fig. 3. Modal resonances of the T_1 wave when proper (soft) background is used.

The specified intermediate background was completely adequate for the T_1 wave when $l \leqslant n \leqslant 50$. With n increasing, the background deteriorates gradually, and by $n \sim 90$ it entirely failed. Here, on the resonance curve of the partial mode its asymmetry becomes more evident, the curve becomes twisted, and with further n increasing in the neighbourhood of the resonance frequency first a typical "hook" is observed instead of a usual resonance curve, and afterwards even a minimum. So, with the specified type of the background for some n value, in order to be sure of the adequacy of the background, one should keep an eye on the solution. The curves $|f_{nm}(x)|$ and $|\bar{f}_{nm}(x)|$ should be computed for every resonance. In the considered x range, they must coincide everywhere except for the range where the resonance occurs. Near the resonance frequency the modulus of the difference $|f_{nm}(x) - \bar{f}_{nm}(x)|$ should have a form of a resonance curve. Theoretically, an effective background can be specified for every n value, but it should be chosen individually [2, 6].

2. Specification of the computational step size for modal resonances

Often high- and low-quality resonances can be situated in the same range of frequency and ordinal number. In Fig. 1, the example of such a situation is shown, where the resonances of shear waves T_0 and T_1 have high Q-factor and those of the zero-order symmetrical S_0 and antisymmetrical A_0 Lamb-type waves have low Q-factor. The curve presented in Fig. 1 has been calculated with the computational step size $l_x = 10/256$. This step size is extremely small for the resonance curves of the S_0 and A_0 waves, and is excessively large for those of the T_0 and T_1 waves.

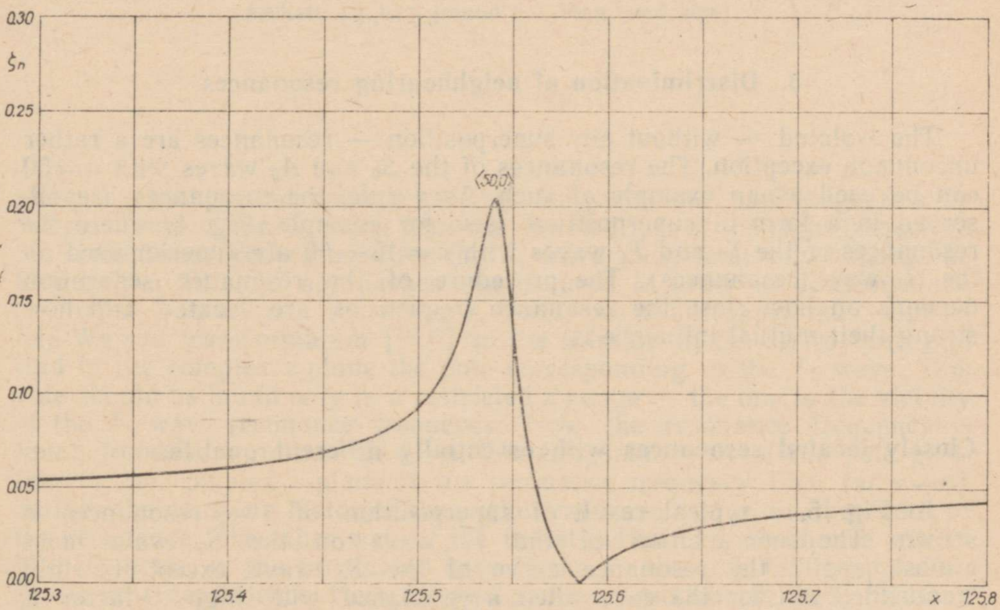

Fig. 4. The same as in Fig. 1, but only for one (the 50th) resonance of the T_0 wave. The computation has been carried out with sufficiently small step size.

The resonances of the S_0 and A_0 waves could be computed with a step size ten times larger without any damage to the description. And vice versa, in order to trace the quality character of the T_0 and T_1 waves, this step size should be reduced approximately one hundred times. In Fig. 4, the resonance curve of the T_0 wave is shown with $n = 50$, computed with the needed step size ($l_x = 0.1/256$).

Since there are frequency and order domains where the resonance components of partial modes change very fast, the complementary computations should be carried out with a smaller step size besides the computation with standard step size in these ranges. The resonances of the T_1 and T_2 waves near the cut-off frequencies can be indicated as an example of such fast changing resonance curves. The resonances of the S_0 wave near the frequency where the overall reduction is compensated by the transverse reduction are also very fast changing (this effect has been described earlier [10, 11]). Generally speaking, the resonance of every peripheral wave has its own specific rate of changing of the frequency, depending on the n value. The computational step size l_x should be specified in such a way that in the active zone of the resonance curve, i. e. situated above the level $1/\sqrt{2}$ of the resonance amplitude, the number of steps will be sufficient.

As the results of computation have shown, in spite of the superposition of two resonances situated near each other, the amplitude of the superposition does not exceed the level of a separated resonance (the formulas for the amplitudes of resonances are given in [12]). Therefore, with correctly specified computational step size and evident (strongly pronounced) form of the resonance curve, the width of (even very narrow) resonance can be found directly from the resonance curve. As can be seen in Fig. 4, with correctly specified step size, the influence of a resonance of another family (with the same ordinal number) can be neglected.

3. Discrimination of neighbouring resonances

The isolated — without any superposition — resonances are a rather uncommon exception. The resonances of the S_0 and A_0 waves with $n=50$ can be used as an example of such. As a rule, the resonances are observed in a form of superposition (see, for example, Fig. 1, where the resonances of the T_0 and T_1 waves with $n=46-60$ are superimposed on the S_0 wave resonances). The procedure of the resonance separation depends on how close the resonance frequencies are located and how strong their mutual influence is.

Closely located resonances with essentially different qualities

In Fig. 5, a typical result of superposition of two resonances is shown. The superposition of the T_1 wave on the S_0 wave does almost “spoil” the resonance curve of the S_0 wave, except its right “tendril”. Even for the case when a resonance with high Q-factor is situated in the active zone of the S_0 wave, its influence could be neutralized by using only the left part of the resonance curve. Here the resonance curve of the S_0 wave is rather wide and its left part is fully sufficient in order to describe the whole resonance curve. Therefore, one could neglect the influence of the T_1 wave resonance on the resonance curve of the S_0 wave. Vice versa, the S_0 wave resonance strongly influences the narrow, with a high Q-factor, resonance of the T_1 wave. One cannot understand where the resonance frequency of the T_1 wave is situated, i. e. whether it coincides with the minimum ($x=86.1$) or with the maximum ($x=86.8$). The computation has shown that in this situation the change of the type of the background, i. e. the utilization of the rigid (or soft) background instead of the intermediate one, does not improve the matter.

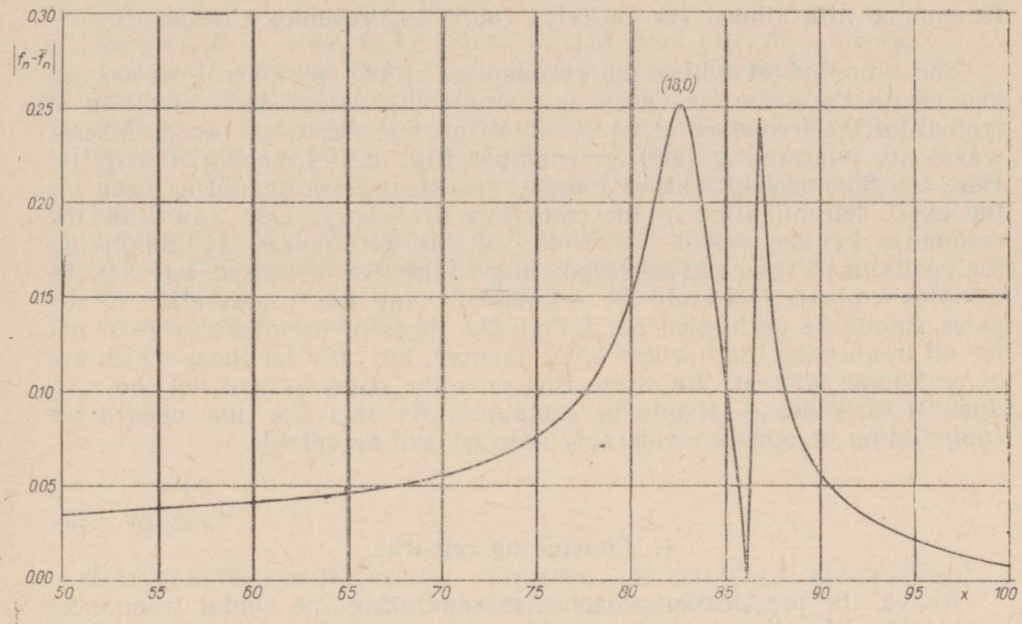

Fig. 5. The character of changing the dependence $|f_n(x) - \bar{f}_n(x)|$ with $n=18$ (An aluminium shell immersed in water [10]; $h=1/10$; $\alpha_0=12°$; $\alpha_*=0°$; $r \to \infty$; the intermediate [9] background has been used here).

Here the background used is entirely adequate: one can judge this by the resonance of the S_0 wave, which is situated just at the right frequency.

Since the procedure used for the T_1 wave fails, some other should be found. Two different approaches can be used in order to determine the resonance frequency of the T_1 wave. First, one could apply the Sommerfeld-Watson transformation [13, 14] to the solution in the series form and find in the complex ν-plane the pole corresponding to the T_1 wave. This pole should be found only in a restricted x range — the one in the vicinity of the T_1 wave resonance frequency. Now the resonance frequency is found from the condition $\mathrm{Re}\,\nu = n$. The imaginary part of the pole position in the complex ν-plane at the resonance frequency $\mathrm{Im}\,\nu$ (at $\nu = n$) defines the damping factor of the peripheral wave and gives the width of the resonance. The other way of the investigation of a resonance with a high Q-factor, in the considered case of the T_1 wave, is the computation of the time dependence. Here the partial form function (in the considered example with $n=18$) should be integrated in a sufficiently long-time, that is narrow in the frequency range, incident pulse. With a specified pulse frequency, one should watch the character of changing the time dependence. When the frequency coincides with that of resonance, the time dependence changes monotonously, and when the frequency coincides with that of antiresonance it will be "swinging", that is, in one turn it will be amplified, and in the other turn attenuated (the time dependences of such kind are shown in Figs. 3.16, 3.14 in [15]). It is easy to find the position of the resonance frequency using the results of computation for some typical values of frequencies. The damping factor (and the width of the resonance) can be found from the damping of the wave on the path of one full turn.

Resonance with almost (or entirely) coinciding resonance frequencies

The situation at which the resonances of two peripheral waves are located on the same frequency is more complicated. This situation is typical for the frequency at which the dispersion curves of two peripheral waves are intersecting (see, for example, Fig. in [16], or Fig. 1 in [17]). Here the Sommerfeld-Watson integral transformation should be used for the exact determination of the resonance frequency. One can find the resonance frequency and the width of the resonances by computing the positions of the poles corresponding to the two discussed waves in the complex ν-plane. It should be mentioned that the computation of the poles should be performed not for all the types of peripheral waves, not for all frequencies, and orders of resonances, but only for those which are of particular interest. We stress that only the poles — and not the residues in the poles — should be computed. By this the time needed for computation becomes considerably reduced and acceptable.

4. Concluding remarks

Above, the practical approaches to separating the modal resonances are outlined for the scattering problem with an obliquely incident plane acoustic wave by a circular cylindrical shell. They can also be used for scattering problems by spherical shells and by solid elastic bodies.

During the computation of the resonance components of partial modes one should watch the adequacy of the background used as well as the computational step size. When the resonance frequencies of the same n order but for different peripheral waves are located near to each other, the standard approach of the resonance scattering theory cannot be used, and it should be substituted by another. Here, in particular, the Sommerfeld-Watson transformation is very effective.

As the experience of the computation and analysis of modal resonances has shown, in the problems of scattering by elastic shells of cylindrical and spherical shape, it is advantageous to solve a model problem before computing the resonance components of partial modes; namely, concerning the propagation of waves in a plane elastic layer. In order to find the resonance frequencies, a "dry" — without any contact with liquid, layer can be used [18]. A layer in contact with liquid should be considered in order to find the resonance width [19]. The solution of the model problem gives an opportunity to understand which difficulties arise at the solution of the main problem. The computation of the model problem is very simple and "cheap", but the gain is rather large, because almost all the features of the problem become clear.

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MODAALSETE RESONANTSIDE LEIDMINE AKUSTILISELT ELASTSETELT KEHADELT HAJUMISE ÜLESANNETES

Statsionaarse hajumisülesande näite varal on esitatud akustiliselt kehalt kaldu langeva tasapinnalise laine poolt tekitatud partsiaalsete moodide resonantskomponentide praktilise arvutamise protseduur. Seoses sellega on vaadeldud kolme küsimust: alustüübi õiget valikut, arvutussammu leidmist ja sageduse järgi lähedaselt paigutatud resonantside määramist.

Наум ВЕКСЛЕР

МОДАЛЬНЫЕ РЕЗОНАНСЫ В ЗАДАЧАХ РАССЕЯНИЯ АКУСТИЧЕСКИХ ВОЛН УПРУГИМИ ОБОЛОЧКАМИ

На примере стационарной задачи рассеяния наклонно падающей плоской акустической волны давления круговой цилиндрической оболочкой излагается процедура практического расчета резонансных компонентов парциальных мод. В этой связи рассматриваются три вопроса: о правильном выборе типа основания, о назначении шага счета по частоте и о различении резонансов, близко расположенных по частоте.