https://doi.org/10.3176/eng.2007.1.01

Proc. Estonian Acad. Sci. Eng., 2007, **13**, 1, 37–47

On the parameters of coastal-trapped topographic waves near the eastern coast of the Baltic Sea

Lembit Talpsepp

Institute of Marine Systems, Tallinn University of Technology, Akadeemia tee 21, 12618 Tallinn, Estonia; talpsepp@sea.ee

Received 19 January 2006, in revised form 24 January 2007

Abstract. In the paper a barotropic model of continental shelf waves with sloping depth of finite width without depth-break at the edge of the shelf is adopted for the region of the eastern part of the Baltic Proper. The dispersion relations and the distribution of the amplitudes of coastal-trapped waves are calculated. It is shown that dispersion relation is stable relative to the characteristic length scale. It was found that shorter waves are more strongly trapped by the coastline. Dispersion curves and the distribution of wave amplitudes are similar to the continental shelf waves in the ocean. The inhomogenities of the scale of 15 km on the water surface in the satellite images may be the result of low-frequency coastal-trapped waves. The current measurements, carried out in the Irbe Strait in June 1995, showed the existence of current oscillations with a period of 44 hours. The model results allow to pose the hypothesis that these oscillations are coastal-trapped waves generated by wind stress variability in the eastern Baltic, from where they propagate northward into the Irbe Strait.

Key words: Baltic Sea, Irbe Strait, coastal-trapped waves, topographic waves, currents.

1. INTRODUCTION

Since 1970s, topographically induced mesoscale variability has been found to be of great importance in the coastal regions of oceans and seas. Continental shelf waves are investigated in many regions of the oceans and seas and the results of these investigations are given in [1–3]. For these topographic waves, with the horizontal scale of some tens to some hundreds of kilometres in the ocean, it is characteristic that they propagate along the coast with the coastline on their right and their amplitudes decrease rather rapidly with the distance from the coastline, i.e. amplitudes are trapped by the topographic structure. Continental shelf waves appear in the coastal regions of the ocean, where the deep ocean borders with a shallow shelf region with a depth-break at the shelf border. Usually the ocean is weakly stratified near the coast and mesoscale processes, including continental shelf waves, can be considered as barotropic waves. In the seas of the World Ocean like the Yellow Sea, the Black Sea, the Baltic Sea etc., water stratification is of greater importance, influencing the appearance of topographically trapped waves and their parameters. This kind of mesoscale waves has been studied by simplified or numerical models in the Baltic Sea by Talpsepp [4], Pizarro and Shaffer [5], Raudsepp et al. [6] and others.

The bottom topography in some regions of the Baltic Sea can be approximated by regions of constant slope near the coast and with flat bottom in the open sea. It is supposed that in some conditions the use of the barotrophic sea model is justified even if the process in nature is not purely barotrophic. In this paper, the parameters of the coastal-trapped waves for the eastern part of the Baltic Sea are studied and corresponding distributions of amplitudes and dispersion curves are found. The dependence of these relations on the characteristic scale of the sea is investigated. Also the possibility of detecting this kind of trapped waves on the satellite images and in direct measurements is discussed.

2. MODEL EQUATIONS

We shall consider barotrophic coastal-trapped waves when the bottom slope has finite width near the coast and flat bottom in the open sea. In the ocean with a continental shelf there exists a step in the depth profile where the bottom slope overgoes to the flat bottom. In our case there is no step present in the depth profile and the bottom slope smoothly turns into flat bottom.

Next we are going to analyse the general case of continuously increasing depth and straight coastline. The theory of barotrophic shelf waves is based on linearized equations of shallow water in a rotating basin. In most regions of the Baltic Sea, in most seasons the stratification is of great importance for the generation and scales of the mesoscale variability. In spite of that, we think that in certain periods the sea water in some coastal regions like the eastern part of the Baltic Proper near Latvian coast is well mixed and barotrophic approach can be used. On the other hand, the Coriolis parameter can be considered to be constant everywhere in the Baltic Sea, thus the β-effect can be excluded.

The linear equations for shallow water may be written as follows:

$$u_t - fv = -g\eta_x,$$

$$v_t + fu = -g\eta_y,$$

$$(Hu)_x + Hv_y = -\eta_t. \qquad (1)$$

Here u and v are depth-integrated velocity components, $\eta(x, y, t)$ is the free surface elevation, f and g are Coriolis parameter and gravity, respectively; the depth is given by the formula $z = -H(x)$. Here the subscript means the derivative with respect to the corresponding argument. We can use the continuity equation in (1) in the given form because $\eta << H$ everywhere except a small region near the shoreline at $x = 0$.

After eliminating u and v from Eqs (1), we get the following equation for η

$$H\Delta\eta_t + H_x\eta_{xt} + fH_x\eta_y - \frac{M}{g}\eta_t = 0, \tag{2}$$

where $\Delta \equiv \partial_{xx} + \partial_{yy}$, $M \equiv \partial_{tt} + f^2$. For waves, propagating parallel to the coast-line, we can seek the solution in the form

$$\eta = F(x)\exp(i(ly + \omega t)). \tag{3}$$

We suggest that always the wave number $l > 0$, but the wave frequency ω may have both positive and negative values, whereas positive frequency $\omega > 0$ corresponds to the waves propagating in the negative direction of the y axis. Substituting (3) into (2), we get the equation for the amplitude $F(x)$:

$$(HF_x)_x + \left[\frac{1}{g}(\omega^2 - f^2) - l^2H + \frac{1}{\omega} flH_x\right]F = 0 \tag{4}$$

in the region $0 < x < \infty$. The corresponding boundary conditions are $Hu = 0$ at $x = 0$, and $\eta \to 0$ in case $x \to \infty$ that may be expressed by the amplitude function F as

$$H(flF + \omega F_x) = 0, \text{ at } x = 0, \text{ implying } |F_x(0)| < C_0, \tag{5}$$

and $F \to 0$ in the case $x \to \infty$.

In [7] the bottom topography was given by the formulae

$$H(x) = dx/L, 0 < x \le L,$$

$$H(x) = D, x > L,$$

where d is the ocean depth near the shelf border, D is the ocean depth and L is the characteristic length. Following [7], we can write the general solution of Eq. (4) as

$$F(x) = Ae^{-lx}L_\nu(2lx), 0 < x \le L,$$

$$F(x) = Be^{-Nx}, x > L. \tag{6}$$

Here $L_\nu(z)$ is the Laguerre polynomial (later $L'_\nu(z)$ means the first derivative of the function $L_\nu(z)$), $\nu = 0, 1, 2, \ldots$.

Parameters A and B are constants. Parameter N is expressed as

$$N^2 = \left[l^2 + \frac{1}{gD}(f^2 - \omega^2) \right].$$

The constant N^2 must be real and positive for trapped waves. From this solution we can see that all low-frequency waves are trapped, because for these waves $|\omega| < |f|$, and $l > 0$.

Continuity of η and Hu at $x = L$ gives two linear equations for finding the parameters A and B. As we are looking for a nontrivial solution (i.e. parameters A and B are not equal to zero), we get the implicit dispersion equation [7] in the form

$$L_\nu(2lL)\left\{ \left[1 + \frac{f^2 L^2}{gD}\left(1 - \frac{\omega^2}{f^2}\right) \right]^{0.5} - \frac{f}{\omega} - \frac{d}{D}\left(1 - \frac{f}{\omega}\right) \right\} + 2\frac{d}{D} L'_\nu(2lL) = 0. \quad (7)$$

The parameter ν is related to other parameters $\sigma = \omega / f$ and $\delta = f^2 L^2 / gD$ by the equation

$$\sigma^3 + [1 + (2\nu + 1)\delta^{-1} lL]\sigma + \delta^{-1} lL = 0. \quad (8)$$

Mysak [7] has studied the special case for the ocean where $d/D << 1$, and got the approximation $L_\nu(2lL) = 0$. We shall study another special case, corresponding to the bottom topography of the eastern Baltic Proper.

3. WAVE PARAMETERS FOR THE EASTERN BALTIC PROPER

The bottom topography for the region near the eastern coast of the Baltic Proper may be approximated with the formulae

$$H(x) = dx/L,\ 0 < x \le L,$$

$$H(x) = d,\ x > L.$$

The parameters in (7) and (8) for the Baltic Sea were calculated as follows: $f = 1.2 \times 10^{-4}\ \mathrm{s}^{-1}$, $L = 15 - 20$ km, $d = 50 - 80$ m. According to these parameters and because $d/D = 1$, we find that $\delta << 1$ and we can use the approximation $L'_\nu(2lL) = 0$ as a dispersion relation instead of Eq. (7).

Next, having studied the dispersion relation for the eastern Baltic Proper, we present the first derivative in the series as

$$L'_\nu(z) = -\nu + \frac{2}{(2!)^2}(-\nu)(-\nu + 1)z + \ldots + \frac{2}{(n!)^2}(-\nu)_n z^{n-1} + \ldots = 0, \quad (9)$$

where $(-\nu)_n = (-\nu)(-\nu+1)\ldots(-\nu+n-1)$. Using (8), we can find the dispersion relation for coastal-trapped waves in the explicit form. After solving the cubic equation (8), taking into account the relationship between parameters $k = lL$ and ν, we obtain three dispersion curves for every mode. Two solutions present the dispersion curves of two fast waves that propagate in different directions with the same phase speed; they are called edge waves. The periods of these waves are ten times or more shorter than the local inertial period. We calculated dispersion curves for the Coriolis parameter $f = 1.2\times10^{-4}\ \mathrm{s}^{-1}$ for the characteristic length scale $L = 20$ km. We found that $\delta = 0.0079$ for the region of the eastern Baltic Sea. Figure 1 shows dispersion curves corresponding to the third solution of the above given system $L'_\nu(2lL) = 0$ and (9) representing subinertial waves.

4. DISCUSSION AND CONCLUSIONS

From the viewpoint of water circulation, the modes of edge waves, which are very fast, can not substantially influence water circulation. The subinertial modes of the coastal-trapped waves are low-frequency coastal-trapped waves, the dispersion curves of which are presented in Fig. 1. These waves can substantially influence water movement in the coastal zone. The dispersion relation, the dispersion curves of which are graphically presented in Fig. 1, was rather stable relative to the changes of the parameter δ; with $\delta = 0.0158$, the dispersion curves varied about 10% in comparison with the case $\delta = 0.0079$.

The amplitude function (6) describes the decrease of the amplitude of the coastal-trapped wave with the distance from the coastline. Figure 2 shows that

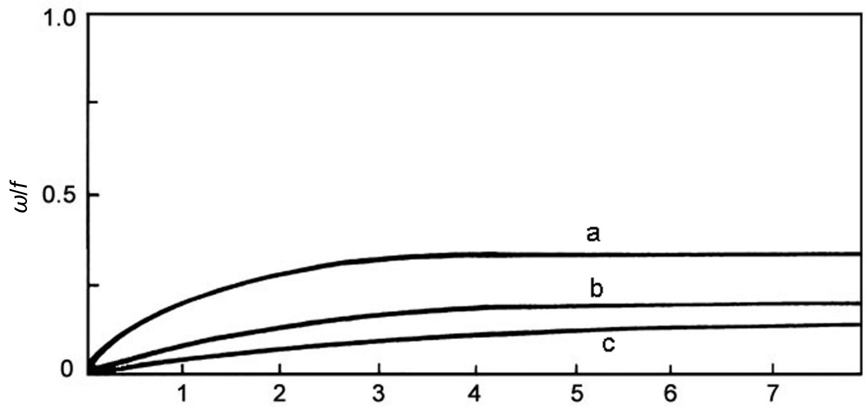

Fig. 1. Dispersion curves of the zero mode (a), first mode (b) and second mode (c) of low-frequency coastal-trapped waves, calculated according to the parameters of the eastern Baltic Sea. On the x axis the dimensionless wavenumber is shown.

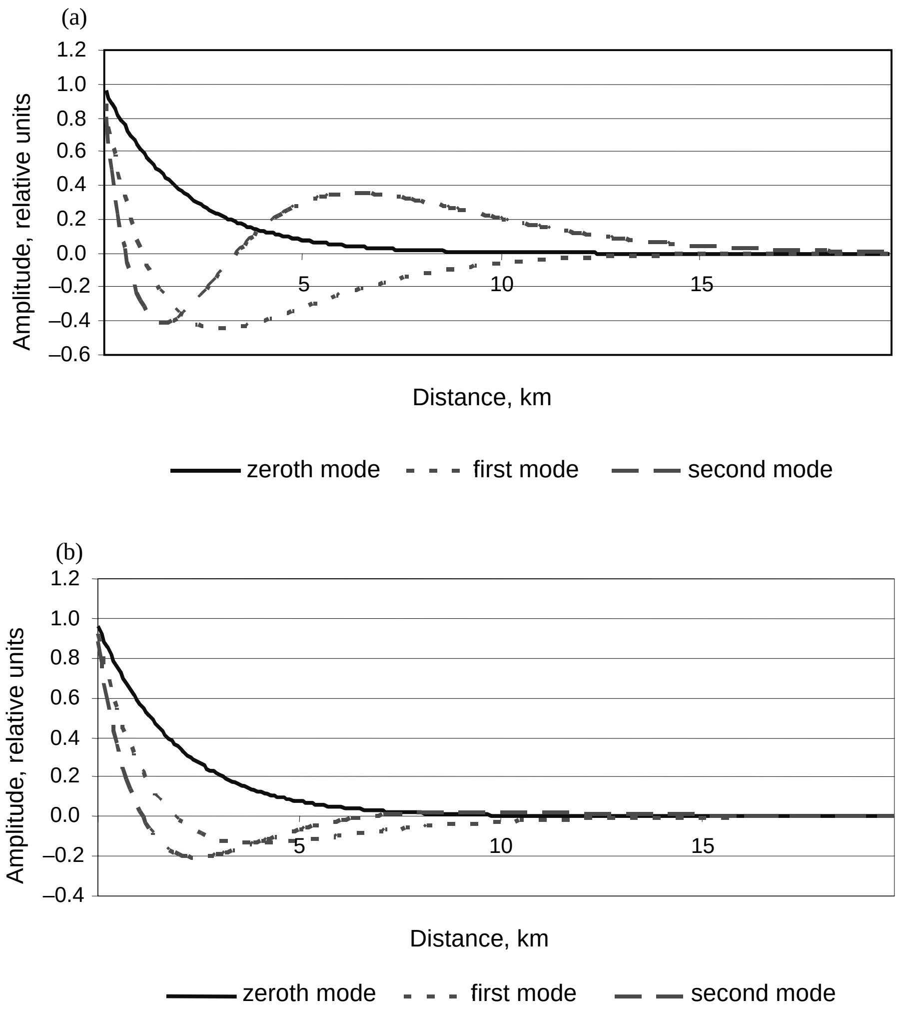

Fig. 2. Dependence of the amplitudes of barotropic coastal-trapped waves on the distance from the coastline: (a) for wavelenght 15 km; (b) for wavelenght 10 km.

amplitudes of all modes decrease rather rapidly with the distance from the coastline. Comparison of Figs. 2a and 2b reveals that shorter waves are much more trapped by the coastline, although the shape of amplitude functions is similar in both cases.

In nature, lower wave modes have most of the energy, so often only the zeroth mode is taken into consideration. The amplitudes of the zeroth mode decrease monotonically with the distance from the coastline. In case of linear waves, the magnitudes of the maximum amplitudes A and B in (6) are not determined by the dispersion relation, they usually depend on the existence of external forces and in case of the Eastern Baltic on wind conditions. Thus, if there exist coastal-trapped waves, their wavelength, period and other parameters like phase speed are determined by dispersion relations (7) and (8). According to Fig. 1, the waves with the wavelength in the range of 15 to 30 km, that is expected spatial scale of mesoscale variability, have periods, which exceed the local inertial period 14 hours for 3.3–4 times.

There are not very many observations of coastal-trapped waves in the Baltic Sea. Two main groups of observations indicate the existence of coastal-trapped waves. First, the observations from satellites in the visible spectrum [8]. Although the use of satellite data is more and more common to follow the phytoplankton blooms [9] or to detect eddies and upwelling in different regions of the Baltic Sea [10], it is seldom used to observe topographic waves. The situation, when the suspended matter is on the sea surface before topographic waves are generated, is seldom recorded. At the same time, satellite images are accepted as evidence of the existence of the upwelling, eddies and topographic waves in many regions of the World Ocean. For example, according to [11], in case of existence of coastal-trapped waves there exist zones with onshore and offshore currents, which create inhomogeneties in suspended matter, if there is suspended matter on the surface before coastal-trapped waves are generated. Thus it is possible to observe coastal-trapped waves in uncloudy weather and when there is a tracer on the sea surface. This situation may occur in nature in algae bloom periods in spring or summer. Sometimes also the distribution of ice may serve as a tracer.

We distinguished at least five inhomogenities on a satellite image of the high-resolution radiometer in the visible range (from 570 to 700 nm) of the eastern Baltic Sea in May 1981. These patches near eastern coast with spatial scale of 15 km correspond to the expected wavelength. The region, where coastal-trapped waves were observed in a satellite image, is presented in Fig. 3. From the satellite image we can see the structure on the sea surface, but not the progress of the structure because a series of good quality satellite images is rather rare. In spite of that, satellite images are widely used by observing processes in oceans and seas, very often supported with direct measurements.

The second way of observing coastal-trapped waves is current measurements by autonomous mooring stations. According to the dispersion relation, coastal-trapped waves in the low-frequency range propagate with the shallower water on their right. According to the map of the eastern Baltic Proper (Fig. 3), coastal-trapped waves propagate to the north. We can see quite a long area with nearly parallel and straight isobathes, which according to theoretical models can generate favourable conditions for coastal-trapped topographic waves. We have made current measurements at two stations in the Irbe Strait in 1994 and 1995 [11,12]. The well-working mooring station P was situated in the nothern part of the Eastern Baltic (Fig. 3). Current velocity directions were measured in every ten minutes by Aanderaa current meters at two levels. We observed periodic components in both current components. The north-east oriented current component, measured in May–June 1995 is presented in Fig. 4, where we can see dominant current oscillations with the amplitude up to 25 cm/s. The amplitude of

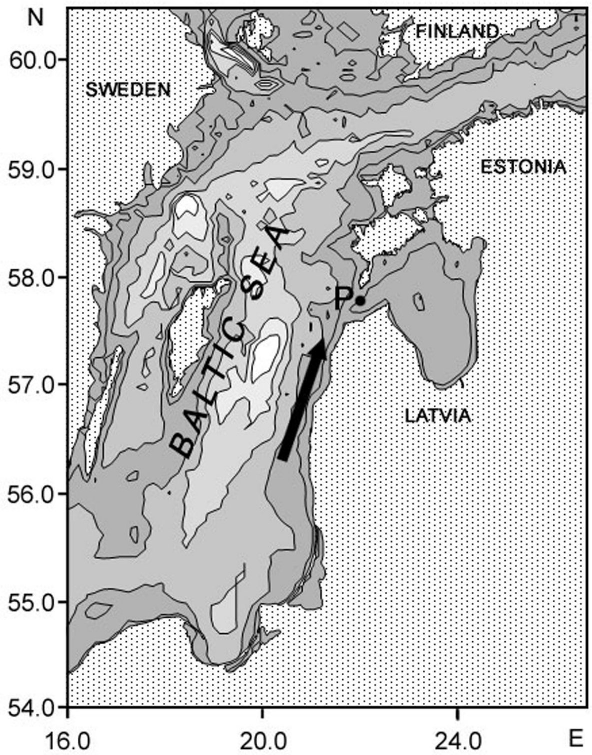

Fig. 3. The location of the mooring station P in the Irbe Strait in the northern part of the eastern coastal area of the Baltic Sea. The direction of propagation of coastal-trapped waves with the shallower water to their right is shown by the black arrow.

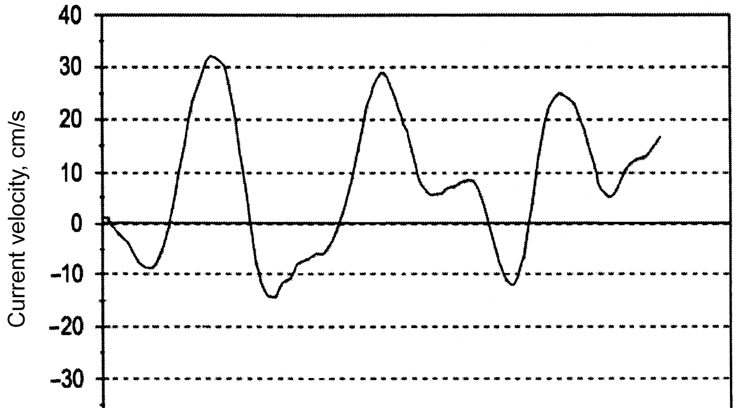

Fig. 4. Alongshore low-passed current component ($T > 24$ h) in June 1995 in Irbe Strait in the northern part of the eastern Baltic Proper. The dominating period is 44 h.

the across-shore current component was about 15 cm/s. By CTD-profiles, acoustic current profiles and by short-time current measurements at the second mooring station, the progressive character of these oscillations was detected [12,13]. We can expect that these oscillations are coastal-trapped waves, generated near the eastern coast and propagating northwards to the mooring station area because parameters of oscillations are close to those predicted by the model of the coastal-trapped waves presented above.

In [11,12], the coherence of these oscillations with wind stress is shown and it is found that oscillations existed also in the Gulf of Riga. It was concluded that oscillations are generated by wind stress variability because during the measurement period many cyclones passed the area with the periodicity of about 42–44 h. Although coherence with wind stress was found, the hydrophysical process, generated by the wind stress, was not established. Due to different spatial scales of the processes in the atmosphere and in the sea, there is no direct relation between these processes; rather the wind stress generates the process in the sea with the same period but with the spatial scale determined by the parameters of the sea. Here we present the hypothesis that these oscillations are generated by wind stress and they originate from the eastern coast of the Baltic Sea. The region, generating coastal-trapped waves, which propagate northward to the Irbe Strait, is shown in Fig. 3. In [11], periodic variability of currents in the Irbe and Virtsu straits were studied. On the basis of the model of Otsmann et al. [14] it was found that the period of 44 h does not correspond to the eigenperiod of the Gulf of Riga, the nearest eigenvalue was found to be about 92 h [14]. All periodic oscillations in the Gulf of Riga as a water basin and also water exchange processes in the straits, induced by local pressure differences, are expected to have periods near the eigenperiods of the Gulf of Riga. The longest period of seiches of the Baltic Sea is measured and calculated to be about 27 h [15]. Thus the period of seiches is much shorter in comparison with the period in our measurements and observed oscillations can not be interpreted in the frame of seiches.

Also the dispersion relation of the coastal-trapped waves supports our hypothesis that these rather strong current oscillations, observed in June 1995 in the Irbe Strait, are coastal-trapped waves propagating northwards along the eastern coast of the Baltic Proper. According to the dispersion relation, presented above, coastal-trapped waves, which have periods in the range of 40–45 h, have a wavelength of about 15–20 km. The calculated phase speed of these waves is about 10–13 cm/s and according to the theoretical distribution of amplitudes, they exist in the near-shore region up to 15–20 km from the coastline.

We have used the barothropic model that is closer to the nature in spring before seasonal thermocline gets strong. Later in summertime the seasonal thermocline becomes more important. Our current measurements in the Irbe Strait were carried out at the beginning of June when the thermocline is still weak and therefore the use of the barothropic model is more justified. Even if the coincidence of the model results and experimental data (both satellite data and current measurements) is occasional, we can conclude that similarly to the ocean, coastal-trapped waves may exist in the eastern Baltic Proper and when they do exist, they have parameters similar to those found above in this paper.

ACKNOWLEDGEMENT

This paper was partly supported by Estonian Science Foundation (grant No. 5869).

REFERENCES

1. LeBlond, P. H. and Mysak, L. A. *Waves in the Ocean*. Elsevier, Amsterdam, 1978.
2. Mysak, L. A. Recent advances in shelf dynamics. *Rev. Geophys. Space Phys.*, 1980, **18**, 211–241.
3. Kamenkovitch, V. M. and Monin, A. C. *Oceanology, vol. 2*. Nauka, Moscow, 1978 (in Russian).
4. Talpsepp, L. On trapped topographic waves in the Baltic Sea. *Oceanology*, 1983, **23**, 928–931 (in Russian).
5. Pizarro, O. and Shaffer, G. Wind-driven, coastal-trapped waves off the island of Gotland, Baltic Sea. *J. Phys. Oceanogr.*, 1998, **28**, 2117–2129.
6. Raudsepp, U., Beletsky, D. and Schwab, D. Basin-scale topographic waves in the Gulf of Riga. *J. Phys. Oceanogr.*, 2003, **33**, 1129–1140.
7. Mysak, L. A. Edgewaves on a gently sloping continental shelf of finite width. *J. Mar. Res.*, 1968, **26**, 24–33.
8. Bychkova, I. and Viktorov, S. Use of satellite data for identification and classification of upwelling in the Baltic Sea. *Oceanology*, 1986, **27**, 158–162 (in Russian).
9. Kahru, M. and Leppänen, J.-M. Cyanobacteria blooms in the Gulf of Finland triggered by saltwater inflow into the Baltic sea. *Mar. Ecol. Progr. Series*, 2000, **207**, 13–18.
10. Lass, H. U. and Talpsepp, L. Observations of coastal jets in the Southern Baltic. *Cont. Shelf Res.*, 1983, **13**, 189–203.
11. Talpsepp, L. Coherent current oscillations and water exchange in the straits of the Gulf of Riga. *Oceanologia*, 2005, **47**, 115–127.
12. Lilover, M.-J., Lips, U., Laanearu, J. and Liljebladh, B. Flow regime in the Irbe Strait. *Aquat. Sci.*, 1998, **60**, 1–13.
13. Talpsepp, L. and Laanearu, J. Eulerian current measurements by RCM-7. In *Estonian Marine Institute Report Series*, 1995, No. 2. EMI, Tallinn, 69–77.
14. Otsmann, M., Astok, V. and Suursaar, Ü. A model for water exchange between Baltic Sea and the Gulf of Riga. *Nord. Hydrol.*, 1997, **28**, 351–364.
15. Tomczak, M. *Introduction to the Oceanography*. 2004, http://www.es.flinders.edu.au/~mattom/IntroOc/lecture10.html

Kaldalähedaste topograafiliste lainete parameetritest Läänemere idaosas

Lembit Talpsepp

Artiklis on Läänemere idakaldale lähedase piirkonna jaoks kohaldatud fikseeritud laiusega põhjakaldega kontinentaalsete šelfilainete mudel, millel puudub ookeanile omane sügavuse järsk suurenemine väljaspool šelfipiirkonda. On arvutatud kaldalähedaste topograafiliste lainete dispersioonikõver ja amplituudide jaotus. On näidatud, et dispersioonikõvera kuju sõltub vähe kaldanõlva laiusest ja et lühemad lained sumbuvad kiiremini kaldajoonest kaugenedes. Dispersioonikõver ja amplituudide ruumiline jaotus sarnanevad ookeani vastavate karakteristikutega. Oletatakse, et kosmosevaatluste tulemusena leitud pinnakihi ebaühtlused mastaabiga ligikaudu 15 km võivad olla tekitatud madalsageduslike kaldalähedaste lainete poolt. 1995. aasta juunis Irbe väinas teostatud mõõtmistega tuvastati hoovuste võnkumine perioodiga 44 tundi. Mudeli tulemused võimaldavad püstitada hüpoteesi, et need võnkumised on tekitatud kaldalähedaste lainete poolt muutuva tuule tingimustes Läänemere idakalda piirkonnas, kust need levivad põhjasuunas Irbe väina.