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WITTGENSTEIN'S TRACTATUS 3.333 AND RUSSELL'S PARADOX

Urmas Sutrop

Institute of the Estonian Language, Tallinn, and the University of Tartu

Abstract. In his 'Tractatus logico-philosophicus', Ludwig Wittgenstein declares that he has solved Russell's paradox. He presents the solution in a prima facie simple formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$". This solution is disregarded both by the Russellians and most Wittgensteinians. In this paper I try to read the above formula and translate it into a class membership language (CML). I show that this formula contradicts Russell's logic. I investigate and compare the different forms of this formula in Wittgenstein's manuscripts and in the different editions of the 'Tractatus'. I also take a look at the different interpretations of this formula in literature. My position is that Wittgenstein created his own transcendental logic that can show self-referential (reflexive) sentences without reflexivity.

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Nobody will wish to assert of the class of men that it is a man.

Gottlob Frege

This is at once clear, if instead of "$F(F(u))$" we write "$(\exists\phi) : F(\phi u) . \phi u = Fu$".

Ludwig Wittgenstein

1. Introduction

Russell's paradox is still topical: it will not go away (Moorcroft 1993). It seems that Moorcroft has rightly pointed out that 'the impact of the Paradox is not so much a problem that must be solved in some way but the abandonment or modification of a principle that was taken to be intuitive and obvious' (1993: 101). The paradox arises by considering the class of all classes which are not members of themselves. Such a class appears to be a member of itself if and only if it is not a member of itself. Ludwig Wittgenstein thought that Russell's paradox vanishes in his 'Tractatus logico-philosophicus' (prop 3.333).[1] In this paper Russell's paradox (contradiction) will be mentioned many times. To be clear, I present here a version of Russell's paradox which Bertrand Russell drafted at a mature age:

> Some classes are members of themselves, some are not, the class of all classes is a class, the class of not-teapots is a not-teapot. Consider the class of all the classes not members of themselves; if it is a member of itself, it is not a member of itself; if it is not, it is (see Garciadiego 1992: xiii).

It is interesting that Post-Tractarian Wittgenstein utters '"the class of cats is not a cat."–How do you know?' and indicates that 'there is a language game with this sentence'. Hence he holds this opinion about the Russellian contradiction which 'would stand like a monument (with a Janus head)' (Wittgenstein 1956, 182e and 131e).[2]

But to our analysis it is important that young Wittgenstein declares that he has solved Russell's paradox – 'Herewith Russell's paradox vanishes' (1922, prop. 3.333(4)). He presents his solution of Russell's paradox in a prima facie simple formula

$$\text{“}(\exists\phi) : F(\phi u) \,.\, \phi u = Fu\text{”}$$

(Wittgenstein 1922, prop. 3.333(4)).

There is an intension in this proposition that the formula solves the paradox better than Bertrand Russell in his theory of logical types. It is curious that Wittgenstein's solution of Russell's paradox is disregarded both by the Russellians and most Wittgensteinians.

At first, Bertrand Russell himself ignored this solution of his paradox. When

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1 This wording – Russell's paradox vanishes – corresponds to the old English translation of Wittgenstein's 'Tractatus' (1922, prop. 3.333). In a modern translation of 'Tractatus' Wittgenstein proposed to "dispose" of Russell's paradox (1961, prop. 3.333). In this paper I prefer the former translation, for the old translation was the authorized version of Ludwig Wittgenstein himself. He discussed every point and comma in the English translation of 'Tractatus' (Wittgenstein 1973a).

2 Wittgenstein's Post-Tractarian views on Russell's paradox–"just don't draw any conclusions from a contradiction"–are not discussed in this paper. For overview see, e.g. Chihara (1977) and Sokuler (1988). It is interesting that in his paper on Wittgenstein's analysis of the paradoxes, Charles S. Chihara ignores Wittgenstein's 'Tractatus' (cf. Chihara 1977).

he, for the first time, read the manuscript of Wittgenstein's 'Tractatus' he sent Ludwig Wittgenstein a letter and queries about the text of the 'Tractatus' on a separate sheet (McGuinnes and von Wright 1990, 107-108. Letter #7 from Bertrand Russell to Ludwig Wittgenstein from 13 August 1919). Here and after this letter, Bertrand Russell did not pay any attention to Wittgenstein's solution of his contradiction. It is typical that Wittgenstein's solution of this paradox is not discussed in the special studies on Russellian contradiction (Quine 1963, Garciadiego 1992, Link 2004) or in the comparison of Whitehead and Russell's 'Principia' and Wittgenstein's 'Tractatus' (e.g. Rao 1998).

Russell discovered his paradox in 1901, originally formulating it in terms of predicates rather than in terms of sets (similar antinomy was discovered by Cesare-Bura Forti already in 1897). Next year Russell wrote to Gottlob Frege of his paradox which showed that the axioms Frege was using to formalize his logic were inconsistent (cf. Frege 1976: 211 ff.) As a result Frege immediately added an appendix to the second volume of his 'Grundgesetze der Arithmetik' (1903) which discussed Russell's contradiction. Russell himself first disputed his paradox in depth in an appendix to his 'Principles of mathematics' (1903) and he developed the idea in his theory of types. The naïve set theory created at the end of the 19th century by Georg Cantor and used by Frege leads to a contradiction. Ivor Grattan Guinnes describes Russell's paradox as a true paradox, a double contradiction, not another neo-Hegelian puzzle to be resolved by synthesis (2000: 311).[3]

One can divide authors who have considered Wittgenstein's solution of Russell's paradox into two groups. Members of the first group tackle Wittgenstein's solution of Russell's paradox very briefly. Often they misinterpret the solution. Philosophers of the second group claim that Ludwig Wittgenstein showed that the ordinary language and the logical symbolism are different ways of thinking. It follows that Russell's paradox is the problem of the ordinary language and Ludwig Wittgenstein created a new logic that contradicts in some way Russell's logic. In this paper I will try to overcome the weakness of ordinary language. For that purpose I will introduce a class-membership language (CML) which helps us understand the formal language written in logical symbols.

After this introduction I will try to read the formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" from the proposition 3.333 in Wittgenstein's 'Tractatus'. I will also introduce some elements of Russell's logic. I ask how this formula corresponds to or contradicts the rules of Russell's logic, that is, is this formula significant in Russellian sense? Then I take a look at some misprints in the different editions of 'Tractatus'. To find out the correct formula I try to follow the historical genesis of this formula. After that I will discuss some interpretations of Wittgenstein's solution of Russell's paradox and show that Wittgenstein created a new transcendental logic that is free from reflexivity. Finally I will take a look at some consequences of the solution.

[3] One can read on the Russell's paradox, possible solutions, and how Russell discovered his paradox, e.g. from Grattan-Guinnes (1978), Coffa (1979), Hill (2004), and Link (2004).

2. Translation into CML

Ludwig Wittgenstein reputedly said in one of his lectures on the foundations of mathematics:

> if we express $\phi(\phi)$ in ordinary words, we don't know what we're talking about [...] it [avoiding expressions like '$\phi(\phi)$'] is just a question of closing certain door, you would only be wary in certain special cases–there is the vast field in which the formula would be all right (Wittgenstein 1976: 229).

Bertrand Russell wrote on the same subject – self-reflexiveness – in one of his unpublished manuscripts: "$\varphi(\varphi x)$ is just as shocking as $\varphi((x), \varphi x)$" (Rodriguez Consuegra 1989: 146).

F. Graf Hoensbroech (1939: 355) holds that

> Russell's paradox rests on the figure "$\phi\phi$", in which "ϕ" is to be a variable function and the juxtaposition of the two "ϕ"s is to express the relation "predicate of".

He thinks that Russell's paradox does not imply that there cannot be propositions in the form '$\phi\phi$' in the object-language, but only in the meta-language (see Langford 1939: 132).

Considering these remarks I try to analyze one of such formulas as '$\phi(\phi)$'– Wittgenstein's solution of Russell's paradox. For that I will try to open a door and trespass upon a self-reflexive field of knowledge. Doing so I translate Wittgenstein's respective formulas into a class-membership language (CML) although Wittgenstein wrote in a proposition: 'The theory of classes is altogether superfluous in mathematics' (Wittgenstein 1922: prop. 6.031).

I will use the CML as follows. Let us begin with 'F(F(u))', starting from 'u'. Our 'u' is equal to 'fx' which is our elementary proposition:

"x is a …", e.g., "x is a cat".

The inner function 'F(u)' in the form 'ϕu' states that:

"x is a y is a member of a class", e.g.,
"x is a cat is a member of a class", e.g.,
"x is a cat is a member of the class of cats".

The outer function 'F(F(u))' in the form 'F(ϕu)' says that:

"x is a y is a member of a class is a member of the class of all those classes which are not members of themselves", e.g.,
"x is a cat is a member of the class of cats is a member of the class of all those classes which are not members of themselves".

Let us say, for the sake of simplicity that 'ϕu' is the class of cats. Then we can read 'F(ϕu)' as follows:

> "the class of cats is a member of the class of all those classes which are not members of themselves"

is equivalent to:

> "the class of cats is not a member of the class of cats".

Giving '(ϕu)' the value:

> "the class of all those classes which are not members of themselves"

we get:

> "the class of all those classes which are not members of themselves is not a member of itself",

which must then be equivalent to:

> "the class of all those classes which are not members of themselves is a member of itself".

This is one possible wording of Russell's paradox (cf. Russell 1908: 222). Thus we have seen that the formula 'F(F(u))' is really about Russell's contradiction and, according to the above, the class of cats is not a cat because the class of cats was not a member of the class of cats.

Now I define the truth-value of a class. This is necessary for our following translations:

> "When we say that a class is false, then we assume that this class has no members and therefore is a null-class of a certain type. But if we take a null-class, we cannot fill this class with unreal entities, fictional objects, and say that this is a class of a certain type. When we say that a class is true, then this class is not a null class, i.e., there is at least one member, but maybe more."

After such preliminary work we can translate and read the formula:

$$\text{“}(\exists\phi) : F(\phi u) \,.\, \phi u = Fu\text{”}$$

(treating the formula as a logical product of two n- and (n + 1)-order functions determining some classes).

The formula offers us two possible readings. First, the truth-value of the logical product of the inner and outer functions is equal to the truth-value of the function that is constituted so that the argument of the inner function has the outer function as an attribute. Second, the truth-value of the logical product of the outer and inner functions is equal to the truth-value of the logical product of the outer function and the function that is constituted so that the argument of the inner function has the outer function as an attribute.

According to the definition of logical product:

$p \,.\, q \,.=.\, \sim (\sim p \vee \sim q)$ Df
where "$p \,.\, q$" is the logical product of p and q

(Whitehead, Russell 1950: 109, ∗3·01) using second reading above our formula becomes:

$$\text{“}(\exists\phi) : \sim (\sim F(\phi u) \vee \sim (\phi u = Fu))\text{”}$$

We can rewrite this formula as a system of two statements:

$$\text{“}(\exists\phi) : \sim (\sim F(\phi u) \vee \sim \phi u)\text{”}$$
$$\text{“}(\exists\phi) : \sim (\sim F(\phi u) \vee \sim Fu)\text{”}$$

We can read this system as follows:

> "There exists at least one (n + 1)-order function so that the (n + 2)-order function with the (n + 1)-order argument, and (n + 1)-order function with the n-order argument and the (n + 2)-order function with the n-order argument are all true and their truth-value is equal to the truth-value of the function which is constituted so that the argument of the (n + 1)-order function, the n-order function, has the (n + 2)-order function as an attribute."

If we let the wording of Russell's paradox echo this reading, these formulations become:

> "There exists at least one class such that this class and the statement that this class is a member of the class of all those classes which are not members of themselves are both true and {*} the statement that a member of this class is also a member of the class of all those classes which are not members of themselves is also true."

And:

> "There exists at least one class such that it is false that either this class is false or the statement that this class is a member of the class of all those classes which are not members of themselves is false and {**} it is false that the statement that a member of this class is also a member of the class of all those classes which are not members of themselves is also false".

If one supposes again that the class that exists is the class of cats, we get:

> "{*} the statement that a cat is a member of the class of all those classes which are not members of themselves is true".

Or:

> "{**} it is false that the statement that a cat is a member of the class of all those classes which are not members of themselves is false".

From here it is obvious that this is a new contradiction, for a cat is a simple and not a complex, i.e., not at all a class; but how do we know that? It seems that we can rewrite our formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" also in another way without affecting its truth-conditions:

$$\text{“}(\exists\phi) : F(\phi u) .\ Fu = \phi u\text{”}$$

Besides it is possible to interpret this formula first as containing two logical products "$F(\phi u) .\ Fu$" and "$F(\phi u) . \phi u$" which both having the same truth-value. Second the formula contains only one logical product "$F(\phi u) . \phi u$" which truth value is equal to "Fu".

3. Russell's logic

According to Russell's theory of logical types the formula

$$\text{“}(\exists\phi) : F(\phi u) . \phi u = Fu\text{”}$$

is meaningless.

At first, since according to the axiom of reducibility:

> If ϕx is *any* function of x, we can not make propositions beginning with "ϕ" or "$\exists\phi$", since we can not consider "all functions" (Russell 1908: 248).

But the above formula is according to Russell's contradiction about "all". On the other hand Wittgenstein writes:

> Propositions like Russell's "axiom of reducibility" are not logical propositions, and this explains our feeling that if true, they can only be true by a happy chance (Wittgenstein 1922: prop. 6.1232).

Second, if anybody can show that this formula is predicative or the axiom of reducibility will not be valid even by a happy chance, according to the axiom of identification of variables, the logical product of two functions

> is only legitimate if the two functions take arguments of the same type, for otherwise their logical product is meaningless (Russell 1908: 247).

The logical product of two propositions is valid

> whenever ϕ and ψ take arguments of the same type (Whitehead, Russell 1950: 109, $*3 \cdot 02$).

In the present case, the logical product is taken from two functions which take n- and (n + 1)-order arguments respectively.

4. The historical genesis of the formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$"

At first I will take a look at some misprints in 'Tractatus' that move from one edition to another. Second, I will look at the historical genesis of the formula. The third paragraph (3) of the proposition 3.333 of Wittgenstein's 'Tractatus' (1922) is:

> This is at once clear, if instead of "F(F(u))" we write "$(\exists\phi) : F(\phi u) . \phi u = Fu$".

In many editions of 'Tractatus' there is a misprint in the first formula in this paragraph. Instead of "F(F(u))", "F(Fu)" is often printed, probably because it is prima facie very impressive to interpret the formula as 'This is at once clear, if instead of "F(Fu)" we write "$(\exists\phi) : F(\phi u) . \phi u = Fu$"'. However, this is highly misleading, for in this case the inner function "Fu" in the first formula "F(Fu))" is equal to the "Fu" in the second formula. But this is a totally new statement and cannot be correct since in the second formula the inner function is "ϕu" and this is not equal to "Fu" on the right of the equal sign or we can write "$(\exists\phi) : F(\phi u) . \phi u = \phi u$" which is an example of a tautology and therefore without sense.

This mistake proceeds from the new English translation of Wittgenstein's 'Tractatus' (1961: prop. 3.333). From this translation the mistake moves to German 'Werkausgabe' (Wittgenstein 1989a: prop. 3.333). In the German critical edition of the 'Tractatus' the mistake is corrected (1989b: prop. 3.333).

A German paperback edition of 'Tractatus' contains another mistake (Wittgenstein 1973a: prop. 3.333). The second formula is given in the form "(∃φ) : F(φu) . ψu = Fu". This leads to a similar error, for "ψ" is the symbol for outer function, and is therefore equal to "F" (cf. Wittgenstein 1922: prop. 3.333(2)). That gives to the second formula the form "(∃ϕ) : F(ϕu) . Fu = Fu" which is a new example of tautology.

To be sure which forms of these two formulas are correct, we must look at the origin and genesis of these formulas. In a manuscript that is known under the name 'Prototractatus', it is said:

> Dies wird sofort klar wenn wir statt „F{F(*f* ξ)}" schreiben
> „(**E**φ) · F{φη} · ϕη = Fη"

(see the Cornell Copy of Wittgenstein's *Nachlaß* (Cornell University Library 1967), vol. 5 (von Wright catalogue # MS 104), f. 46 (90) or Wittgenstein 1971: facsimile part, prop. 3.201731). In Wittgenstein's manuscript, this symbol "**E**" (exists) is actually an overwritten symbol "F" (function).

In the Vienna typescript of Wittgenstein's 'Tractatus' it is said:

> Dies wird sofort klar, wenn wir statt „F(F(fx))" schreiben
> „(Eg) : F(gu) . gu = Fu"

(see resp. typescript, von Wright catalogue # TS 203, prop. 3.333(3)). The text of this paragraph in the first printed edition of Wittgenstein's 'Tractatus' (the Ostwald-Ausgabe):

> Dies wird sofort klar, wenn wir statt „F(F(fx))" schreiben
> „(Eg) : E (gu) . gu = Fu".)

corresponds exactly to the Vienna typescript of 'Tractatus' (Wittgenstein 1921: prop. 3.333). The second formula, however, contains two misprints. The second "E" must be read "F" (the symbol for outer function), and there is an extra bracket outside the punctuation.

Starting from the first German-English parallel edition it has been said:

> This is at once clear, if instead of "F(F(u))" we write
> "(∃ϕ) : F(ϕu) . ϕu = Fu".

(Wittgenstein 1922: prop. 3.333). It is the same in the final corrected and authorized version of Wittgenstein's 'Tractatus' (Wittgenstein 1933: prop. 3.333).

In the course of preparations for the first English translation of 'Tractatus', Wittgenstein comments on the proposition 3.333 only once, saying that a comma must be outside the quotation marks (see Wittgenstein 1973b, 63). This leads to the opinion that Wittgenstein agreed entirely with the new form of proposition 3.333. As for this proposition, there are two principal changes in the text compared with the 1921 edition of the 'Tractatus'. Instead of "Eg" there stands "∃ϕ"; and instead of “fx” there stands “u”. The latter case means only that “fx = u”. This change does not make any change in any (logical) sense in this proposition. The former is more complicated.

It is not clear who actually made these changes – the translators, the editor or Wittgenstein himself. Since it is unclear who made the change from “E” to “∃”, it is difficult to decide whether there is any difference at all in the meaning.

In his ‘Notebook’, Wittgenstein writes:

> „(∃x)” sagte „und diese ist A”

(see Wittgenstein 1989a: 102 (17.10.14)) and in ‘Tractatus’ Wittgenstein writes:

> “there is one and only one x, which . . . .”: and this x is a

(see Wittgenstein 1922: prop. 5.526), but later, in 1929, Wittgenstein remarks:

> [Dies bedeutet (E.)
> es gibt nur]

(see the Cornell Copy of Wittgenstein’s *Nachlaß* (Cornell University Library 1967), vol. 8 (von Wright catalogue # MS 106), f. 16).

If “(E.)” means ‘There is only (one and no more)’, does then the existential quantifier “(∃.)” mean ‘There is at least one, but maybe more’? There is also the following difference in the symbolism in some cases – “E” as opposed to “E!” The latter denotes the existence of a predicative function and the former denotes the existence of a non-predicative function. Although in the case of Wittgenstein’s ‘Tractatus’, proposition 3.333, there is no ground for such opposition, we cannot decide what Wittgenstein means by his “E” and “∃”.

The fourth paragraph of proposition 3.333: ‘Herewith Russell’s paradox vanishes’ is added to ‘Tractatus’ in a later stage of completing the manuscript. This paragraph is absent in the ‘Prototractatus’, in a notebook from the year 1918 (see Wittgenstein 1971), but is typed in the Vienna typescript of ‘Tractatus’ from the same year.[4]

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[4] An entry in a Philip Jourdain’s notebook may be crucial for dating the sentence “Herewith Russell’s paradox vanishes: “On April 20 1909, Russell came and had lunch and tea […] Russell said that the views I gave in a reply to Wittgenstein (who had ‘solved’ Russell’s contradiction) agree with his own” (see Grattan-Guinness 1977: 112-118). Grattan-Guinness doubts the correctness of the exact date of this entry. It is quite possible that Philip Jourdain had made an error writing the date ‘On April 20 1909’, and the year should be read ‘1919’. This suggestion is based on the supposition that at that time, in spring 1919, it was already possible to know something about the text of Wittgenstein’s ‘Tractatus’, and therefore also about the quotation in which Russell’s paradox (contradiction) vanishes.

5. How do philosophers interpret Wittgenstein's solution of Russell's paradox?

The first group of the Wittgenstein commentators who consider his solution of Russell's paradox pass over the solution very briefly and superficially (e.g. Allaire 1960, Goldstein 2002: 432). Some unfortunate misinterpretations are due to misprints in some editions of Wittgenstein's 'Tractatus'. In one treatise, for example, it is simply said 'Russell's paradox vanishes in notation which shows that a function cannot be its own argument' (Plochmann, Lawson 1962: 122), or in another treatise, for example, it is noted that 'in six sentences (3.332 - 3.333) Wittgenstein claims to have reproduced the whole of the Theory of Types' (Griffin 1964: 139). James Conant (2000: 212-213), for example, holds that

> the whole point of §§3.3-3.344 of the *Tractatus* is that the identity of the object referred to by a name is only fixed by the use of the name in a set of significant [*sinnvolle*] propositions.

Of course, Conant expresses the critique of Russellian doctrines in Wittgenstein's 'Tractatus' in the quoted passage. His position is similar to those who argue that saying and showing are different or the (visually) same sign can show different objects or propositions.

In a historical study of Wittgenstein's solution of paradoxes, his solution of Russell's paradox is only quoted (Dumitriu 1974: 233). The following analysis in that paper does not touch the problem of the expanded formula. The discussion there is focussed on the prototype 'F(F(fx))'. Dumitriu uses the negation for analyzing self-reflexiveness in the form '$\sim\varphi(\varphi)$' (cf. Behmann 1931). Many other authors, e.g. Davant (1975), have similar views. Davant writes that if Russell's paradox may be disposed without the help of a theory of types, the theory is superfluous. If we rewrite the formula 'F(F(fx))' in the form '$\psi[\phi(fx)]$' the inner and the outer functions are "markedly different" and the meanings of the signs ψ and ϕ are not the same because the forms are different (cf. Davant 1975: 106). It follows that the identical form F of the inner and outer functions causes the contradiction.

On the other hand, Ostrow points that there is no paradox. "In 'F(F(fx))', the first 'F' and the second will not have the same meaning, since, to use Russellian terminology, the first 'F' ranges over propositional functions of type n, while the second ranges over functions of type n + 1" (Ostrow 2002: 66-67). In this case the inner and the outer functions play different roles and the common letter F denoting both functions is not confusing at all. This is very promising approach, but unfortunately the Wittgenstein's formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" is not discussed at all in this paper.

Now I will show what happens if one uses the edition of 'Tractatus' that contains misprints. For example, Daniels and Davidson (1973: 237) argue:

> For Wittgenstein, proposition picture, they don't name or designate at all. Wittgenstein objects to Russell on similar grounds: that *'Fu'* serves *both* as a name and as a proposition (3.333).

It is very difficult to understand what is meant by 'Fu' here, for these authors use the new translation of Wittgenstein's 'Tractatus' (1961) which contains a misprint. In proposition 3.333 F(Fu) is printed instead of the correct form F(F(u)). This is most misleading. The same concerns also the following:

> If instead of F(Fu) we write $(\exists\phi) : F(\phi u) \,.\, \phi u = Fu$ then *it comes out* that there is $\phi u \neq u$, and $F(Fu) \neq Fu$. This is the logico-syntactical solution of Russell's paradox (Scheier 1991:102–103, # 3.333; my translation – US).

Also, it is very difficult to understand how *it comes out* that there is $\phi u \neq u$, and $F(Fu) \neq Fu$, if instead of this F(Fu) we write the expanded formula.

Only very few papers are entirely dedicated to Tractarian proposition 3.333 (Ishiguro 1981 and Jolley 2004). Ishiguro's article considers Wittgenstein's theory of logical types and the solution of Russell's paradox, but the formula in question is mentioned only in one footnote:

> The way in which Wittgenstein reformulates his problem in the latter half of 3.333 is misleading. He writes that the reason why in 'F(Fu)' the outer 'F' and the inner 'F' cannot be the same becomes clear if we try to rewrite it as $(\exists\phi) : F(\phi u) \,,\, \phi u = Fu$. This is drawing attention to someone confusing first-order and second-order predicates, whereas the original example 'F(F(fx))' was based on a confusion between second- and third-order predicates (Ishiguro 1981, 59, note 16).

This is a very unfortunate note, because first Ishiguro, too, uses the new translation of Wittgenstein's 'Tractatus', where proposition 3.333 contains some misprints. Second, in this note the logical product in the formula "$(\exists\phi) : F(\phi u) \,,\, \phi u = Fu$" is substituted, because the symbol – point [**.**] between '$F(\phi u)$' and 'ϕu' is replaced by another symbol – comma [**,**]. This is a new statement. Third, Ishiguro claims that there are first- and second-order predicates in the formula, but the prototype 'F(F(fx))' (from 3.333(2)) contains second- and third-order predicates. This is also misleading, for in this context 'u = fx'. What is important is that in this place n- and (n + 1)-order predicates must be confused. The formula 'F(F(fx))' is really about [(n + 2)((n + 1)n)]-order predicates, and the formula "$(\exists\phi) : F(\phi u) \,.\, \phi u = Fu$" is about $[(\exists f_{n+1}) : f_{n+2}\, x_{n+1} \,.\, f_{n+1}\, x_n = f_{n+2}\, x_n]$-order predicates.

Joachim Bromand describes a general situation where a proposition is (n+1)-order and it says something on n-order proposition but nothing on the propositions which order is greater than n. To avoid paradoxes one should reject concepts that cover all orders and types. Instead one should use hierarchical predicates, e.g. the hierarchical truth-predicate: true 1st order predicate, true n-order predicate, true (n+1)-order predicate, etc. Bromand concludes that the hierarchical truth-concept becomes so fragmented that it is not possible to construct any semantic language for the theory of types (Bromand 2000: 24). For that reason we should avoid the hierarchical relativization of the truth-concept.

Ivor Grattan-Guinness passes over Wittgenstein's solution to Russell's paradox very briefly. He argues that Wittgenstein solved Russell's paradox by preventing a propositional function from being an argument of itself (2000: 439).

It is characteristic of the philosophers of the first group that they interpret Wittgenstein's solution to Russell's paradox in the way that the formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" shows the errors and impossibility of Russell's theory of types (cf. Finch 1971: 90-91).

The second group of philosophers is more serious. They try to interpret Wittgenstein's solution of Russell's paradox. Some of them claim that Ludwig Wittgenstein showed that the ordinary language and logical symbolism are different ways of thinking. Then it follows that Russell's paradox is the problem of the ordinary language and Ludwig Wittgenstein created a new logic that in some way contradicts Russell's logic.

One of the first serious commentators, Max Black, in his famous 'Companion to Wittgenstein's "Tractatus"' tried to explain Wittgenstein's solution assuming that the inner and outer function "F" in "F(F(u))" behaves differently:

> In supplying the expanded formula ["$(\exists\phi) : F(\phi u) . \phi u = Fu$"] on the second line of (3) [prop. 3.333(3)] W[ittgenstein] apparently wishes to show that the outer F obeys different laws of transformation from those of the inner F [in "F(F(u))"] (and hence they must signify different functions). (Black 1964, 149).

Earlier Feibleman expressed the same in a commentary to Wittgenstein's 'Tractatus':

> Manipulations can be conducted in the world of logic, not manipulatable in the world of fact [...] we have in addition to our epistemological realism a metaphysical realism (Feibleman 1958: 75 - 76, # 3.333).

Unfortunately nothing is said about Wittgenstein's formula and Russell's paradox in this commentary. Another author, H. O. Mounce, after a short explanation assumes:

> In short, a theory of type is entirely unnecessary. For in a correct symbolism the problem with which Russell wishes to deal simply will not arise. It will disappear in the very operation of the signs (Mounce 1981: 56).

This interpretation is in accordance with Eli Friedlander's interpretation, which states 'Only the letter "F" is common to the two functions, but the letter in itself signifies nothing .... We must never confuse the level of signs and the level of symbols" (Friedlander 2001: 84). Also Daniels and Davidson argue the same although they used the incorrect formula for their analysis (1973: 237). Paul Livingstone explains that Russell's paradox arises from a notational confusion, for in F(F(fx)) the outer F and the inner F are only the same orthographic signs, but they are two different symbols (2004: 43).

Kelly Dean Jolley argues in his recent paper dedicated to the Tractarian proposition 3.333 that

> Wittgenstein has not logically transubstantiated the Theory of Types, turning it from expressible into inexpressible, from what can be said into what can only be shown, but that he has indeed made the Theory vanish (Jolley 2004: 282).

He explains the Wittgenstein's idea in the following manner:

> … the way the symbols combine is visible in the symbols themselves. The outer function has, so to speak, *two* blank spots while the inner function has only one, So there s no question of the outer and inner functions being switched in a type-transgressing way, nor is there any question of the sameness and difference of the two functions: the two functions are *visibly* different … (Jolley 2004: 285).

> ... the two signs are different symbols. Given this, there is no danger of Russell's paradox arising. Whatever the signs may lead us (mistakenly) to believe, there will be no sentence in which a function acts as its own argument (ibid. 290).

Jolley concludes that in F(F(fx)) two "F" signs are different symbols for, the first, i.e. the outer "F" sign has two blank spots, and the second, i.e. the inner "F" sign has only one. But if we hold that the letter "F" itself signifies nothing, this explanation does not help us to understand Wittgenstein's thought. Jolley's argument is very similar to the Davant's position discussed above – "F" does not have the same meaning in the two occurrences (cf. Davant 1975: 106). Both Davant and Jolley relay on the Tractarian propositions 3.32 and 3.321. Wittgenstein claims that the sign is the part of the symbol perceptible by the senses (3.32) and two different symbols can have the sign in common; it follows that the common sign signifies in two different ways (3.321).[5]

Wilhelm Vossenkuhl (2000) points out that we should make a distinction between Saying and Showing. One can say something through sentences, i.e. through language and one can only show what is unsayable through sentences. Wittgenstein's solution is not philosophical or linguistic but logical. A symbol is allowed to express only what it can express. The Russell paradox is a linguistic one. It belongs to the realm of Saying, but Wittgenstein's solution is logical, it belongs to the realm of Showing. Russell's paradox (his Theory of Types) contains self-reference and for that reason is circular. Wittgenstein's solution in his formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" contains no self-reference (reflexivity). The

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[5] Russell holds another position and accepts the identity of (visually) different signs: $\dot{\varepsilon}\varphi(\varepsilon)$ and $\dot{\varepsilon}\psi(\varepsilon)$ are *identical* when φx and ψx are *equivalent* for all values of x (Russell to Frege on 10.7.1902; Frege 1976: 220).

symbols show the same sentence that lies behind Russell's paradox, but without saying it. Wittgenstein's logic is not a theory but a reflexion of the world. His logic is transcendental (see, e.g. Wittgenstein 1973a: 169, # 6.13).

Nevertheless, we have an access to what is showed by Wittgenstein's transcendental logic. Using CML one can say in "ordinary" language what is only showed by symbols of logic.

6. Consequences

Eddy Zemach appealing to propositions 3.332 and 3.333 in Wittgenstein's 'Tractatus' assumes:

> if God is the meaning of the world, i.e. what the world represents, God must be a fact which is neither in nor outside the world. But this again is impossible (Zemach 1966: 362).

If we use the formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" for proving the existence of God, whereas "u" would mean "x is a perfection" and "ϕu" would mean "the class of all perfections", the new argument sounds:

> "There exists at least one class of all perfections so that this class of all perfections and the statement that this class is a member of the class of all those classes which are not members of themselves are both true and the statement that a perfection is also a member of the class of all those classes which are not members of themselves is also true".

This would add:

> "The most perfect Being has *all* perfections, existence is a perfection; therefore the most perfect Being exists"

to the following:

> "A perfection, say, existence is a member of the class of all those classes which are not members of themselves"

which must also be true. According to the interpretation that Wittgenstein's transcendental logic used in the formula "$(\exists\phi) : F(\phi u) . \phi u = Fu$" shows no self-reference (reflexivity), Eddy Zemach's dilemma gets another solution. God must be a fact which is neither in nor outside the world.

In his essay 'On a property of a perfect being' about Wittgenstein, Morris Lazerowitz uses 'the Anselmic conception of a Being than which a more perfect is inconceivable. This is the idea of a most powerful Being than which a more powerful is inconceivable, a Being which has power to an infinite degree'. He also points out that the Anselmic conception uses the idea, later developed by Georg Cantor, of the completed, or consummated, infinite, the idea of a set all the theoretically possible elements of which are actual elements (1984: 218).

Finally he concludes that 'It is not hard to see why not even God can run through an infinite series or sequences of terms, and why neither God nor we can contemplate an infinite totality of elements. […] It is linguistic usage that prevents the existence of an infinite extension, or its grasp by a mind. And also it is linguistic usage which prevents counting an infinite number of objects' (Lazerowitz 1984: 224).

The final sentence of Wittgenstein's 'Tractatus' says that we can speak only about these cases which come into existence and we cannot speak about those cases which do not come into existence. The last instance can only be shown.

I hope I was able to open a door and walk into an unknown, uncommon landscape. My purpose was to translate some notions from Wittgenstein's transcendental logic into the class-membership language (CML). Although this is not the language which an ordinary man uses, these translations show that it is possible to speak about self-referential functions that can be shown without reflexivity using Wittgenstein's transcendental logic that employs words, not only signs or symbols. And what is sayable (not only showable) is also thinkable. I also showed that Wittgenstein's logic contradicts Russell's logic in many ways, but Wittgenstein's logic allows us to overcome the disadvantages of the Theory of Types.

When we try to understand better what is at once clear, if instead of "$F(F(u))$" we write "$(\exists\phi) : F(\phi u) . \phi u = Fu$" we must investigate the landscape of the manuscript of 'Prototractatus', for the connections between the propositions in their natural order, in their Prototractarian order, and in their order which corresponds to the 'Tractatus' give us at least three different texts. So we can see new horizons.

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Address:
Urmas Sutrop
Institute of the Estonian Language
Roosikrantsi 6
10119 Tallinn, Estonia
Tel: +372 6177 501
E-mail: Urmas.Sutrop@eki.ee

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