Effect of structural parameters on the vibrational response of a visco-elastic rectangular plate with clamped ends

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Abstract. A mathematical model for predicting the vibrational response of a non-homogeneous visco-elastic rectangular plate was developed to assist design engineers and researchers. In the presented model, thermally induced vibrations of a four-sided clamped rectangular plate of non-uniform thickness is discussed. Non-homogeneity in the material is characterized exponentially in Poisson’s ratio while temperature variation is considered bi-parabolic. A boundary value fourth order partial differential equation of motion is formulated for the parabolic tapered rectangular plate. Visco-elastic properties of the material are of Kelvin type, and deflection is considered small and linear. This paper focuses on the effects of structural parameters, i.e. thermal gradient, taper constant, aspect ratio, and non-homogeneity constant on the vibrational behaviour of rectangular plates. The Rayleigh–Ritz method is used to obtain results for the time period and deflection for the first two modes of vibration. Comparison of the results of the present paper with others available in the literature is visualized with the help of graphs.

Key words: visco-elastic, non-homogeneous, structural parameters, deflection, thermal gradient.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>length of the rectangular plate, m</td>
</tr>
<tr>
<td>b</td>
<td>breadth of the rectangular plate, m</td>
</tr>
<tr>
<td>x, y</td>
<td>co-ordinates in the plane of the plate</td>
</tr>
<tr>
<td>h</td>
<td>thickness of the plate, m</td>
</tr>
<tr>
<td>M_x, M_y</td>
<td>bending moments, N·m</td>
</tr>
<tr>
<td>M_{xy}</td>
<td>twisting moment, N·m</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus, N/m²</td>
</tr>
<tr>
<td>G</td>
<td>shear modulus, N/m²</td>
</tr>
<tr>
<td>\nu</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>D_t</td>
<td>flexural rigidity, N·m</td>
</tr>
<tr>
<td>\rho</td>
<td>density of the plate material, kg/m³</td>
</tr>
<tr>
<td>\eta</td>
<td>visco-elastic constant, N·s/m²</td>
</tr>
<tr>
<td>\psi(t)</td>
<td>time function, s</td>
</tr>
<tr>
<td>\alpha</td>
<td>thermal gradient</td>
</tr>
<tr>
<td>\beta</td>
<td>taper constant</td>
</tr>
<tr>
<td>\alpha_i</td>
<td>non-homogeneity constant</td>
</tr>
</tbody>
</table>

1. INTRODUCTION

The designs of complex structures, i.e. jet engines, helicopter yokes, submarines, etc., are based on the vibrational analysis of the structural system. Due to the variability in the mechanical prospective of the structure, the vibrational behaviour of the structure is affected. Therefore, it becomes necessary to analyse the behaviour of structural systems for the preliminary phase of designing structures so that their hydrodynamic performance and stability during navigation and operations can be significantly improved.

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Tapered visco-elastic plates are commonly used in various engineering applications and structures, e.g. the aerospace industry, missiles, etc., under the influence of elevated temperature. Since elevated temperature directly affects the mechanical properties of the material of plates, the effect of temperature variations can not be neglected. Hence, active vibration control in engineering structures and systems has attracted the attention of mathematical physicists and design engineers for many years.


In the present model, the authors analysed the vibrational behaviour of a non-homogeneous rectangular plate. It is assumed that the thickness of the rectangular plate varies parabolically in \( x \)-direction. The Rayleigh–Ritz technique is used to determine the frequency equation of the plate. The time period and deflection at different points for the first two modes of vibration are obtained for various values of structural parameters.

2. GEOMETRY OF THE PLATE

In the present study, a rectangular plate of varying thickness in one direction \( h(x) \), exponential varying Poisson’s ratio \( \nu(x) \), and constant density \( \rho \) is investigated as shown in Fig. 1. Plate OABC is assumed to be placed in such a way that point O is the origin of the \( xy \)-plane and sides OA and OC overlap on \( x \)- and \( y \)-axis, respectively. The domain of the plate in the \( xy \)-plane is \( 0 \leq x \leq a \) and \( 0 \leq y \leq b \), where \( a \) and \( b \) are the length and breadth of the plate, respectively.

![Fig. 1. Rectangular plate in Cartesian coordinate (xy-plane).](image-url)
3. DIFFERENTIAL EQUATION OF MOTION

The equation of motion of a visco-elastic plate of variable thickness is [6]

\[ M_{xx,xx} + 2M_{xy,xy} + M_{y,yy} = \rho \nu w_{xx}, \]  

(1)

where

\[ M_x = -\bar{D}D_1(w_{xx} + \nu w_{yy}), \quad M_y = -\bar{D}D_1(w_{yy} + \nu w_{xx}), \quad M_{xy} = -\bar{D}D_1(1-\nu)w_{xy}. \]  

(2)

Here, ',' denotes partial differentiation with respect to the following suffix.

Using Eq. (2) in Eq. (1), one gets [3]

\[ \bar{D}[\nu^2(D_1\nabla^2 w) - (1-\nu)\phi^4(D_1, w)] + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \]  

(3)

where \( \phi^4(D_1, w) = \frac{\partial^4 D_1}{\partial x^4} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 D_1}{\partial x^2 \partial y^2} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \) is the die operator and \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is the Laplacian operator.

The solution of Eq. (3) can be assumed as

\[ w(x, y, t) = \phi(x, y)t. \]  

(4)

Substituting Eq. (4) into Eq. (3), one obtains

\[ \left[ D_1 \left( \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^2 \phi}{\partial x^2 \partial y^2} \frac{\partial^2 \phi}{\partial x^2} \right) + 2 \frac{\partial D_1}{\partial x} \left( \frac{\partial^3 \phi}{\partial x^3} + \frac{\partial^3 \phi}{\partial y^2 \partial x} \right) + 2 \frac{\partial D_1}{\partial y^2} \left( \frac{\partial^3 \phi}{\partial y^3} + \frac{\partial^3 \phi}{\partial y^2 \partial x} \right) \right] + 2 \frac{\partial^2 D_1}{\partial x^2} \left( \frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} \right) + 2 \frac{\partial^2 D_1}{\partial y^2} \left( \frac{\partial^2 \phi}{\partial y^2} + \nu \frac{\partial^2 \phi}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x^2 \partial y} \frac{\partial^2 \phi}{\partial x^2} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} \right) \right] \rho h = -\dot{\psi} \bar{D}\phi, \]  

(5)

where the dots denote differentiation with respect to \( t \).

Both sides of the preceding equation are independent. Therefore, it is satisfied if both sides are equal to a positive constant, i.e. \( \zeta^2 \). In this case, one gets

\[ D_1 \left( \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^2 \phi}{\partial x^2 \partial y^2} \frac{\partial^2 \phi}{\partial x^2} \right) + 2 \frac{\partial D_1}{\partial x} \left( \frac{\partial^3 \phi}{\partial x^3} + \frac{\partial^3 \phi}{\partial y^2 \partial x} \right) + 2 \frac{\partial D_1}{\partial y^2} \left( \frac{\partial^3 \phi}{\partial y^3} + \frac{\partial^3 \phi}{\partial y^2 \partial x} \right) \]  

\[ + 2 \frac{\partial^2 D_1}{\partial x^2} \left( \frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} \right) + 2 \frac{\partial^2 D_1}{\partial y^2} \left( \frac{\partial^2 \phi}{\partial y^2} + \nu \frac{\partial^2 \phi}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x^2 \partial y} \frac{\partial^2 \phi}{\partial x^2} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} \right) \right] - \rho \zeta^2 h = 0 \]  

(6)

and

\[ \dot{\psi} + \zeta^2 \bar{D}\phi = 0. \]  

(7)

Eq. (6) and Eq. (7) are differential equations of motion and time function for a visco-elastic rectangular plate, respectively.

Here, \( D_1 \) is flexural rigidity of a rectangular plate [14], i.e.

\[ D_1 = \frac{Eh^3}{12(1-\nu^2)}. \]  

(8)
4. LIMITATIONS

Assessment of vibrations of structures is very much complicated due to the variability in structural parameters. Therefore, some restrictions are required for predicting the vibration response of structures. The authors assumed the following limitations in the present model for further investigation:

(i) Most structures are usually worked in the presence of high temperature. Temperature distributions in any structural system vary from point to point. Due to variations in the temperature field, vibrational properties of the structure vary significantly. So, it becomes necessary to analyse the temperature effect on the vibrations of structures or systems.

In the present model, the authors assumed bi-parabolic variation in the temperature field as follows:

\[ \tau = \tau_0 \left( 1 - \frac{x^2}{a^2} \right) \left( 1 - \frac{y^2}{b^2} \right). \]  

The temperature dependence of the modulus of elasticity for most engineering materials can be expressed as:

\[ E = E_0 (1 - \gamma \tau), \]  

where \( E_0 \) is the value of Young’s modulus at reference temperature, i.e. \( \tau = 0 \), and \( \gamma \) is the slope of the variation of \( E \) with \( \tau \). After substituting \( \tau \) from Eq. (9), Eq. (10) becomes

\[ E = E_0 \left[ 1 - \alpha \left( 1 - \frac{x^2}{a^2} \right) \left( 1 - \frac{y^2}{b^2} \right) \right], \]  

where \( \alpha = \gamma \tau_0 \).

(ii) To fulfil the twofold requirement of safety and economy, plates of variable thickness are commonly used in engineering applications, e.g. to make blades of turbines, bridge plates, fins of plane, etc. In order to investigate the effect of tapering on the vibrational behaviour of plates, the authors assumed parabolic tapering in this model:

\[ h = h_0 \left( 1 + \beta \frac{x^2}{a^2} \right), \]  

where \( h_0 \) is the thickness of the plate at \( x = y = 0 \).

(iii) Physical properties of materials, i.e. their density, Poisson’s ratio, etc., vary point to point in case of non-homogeneity is present in the material. In the present model, the authors assumed exponential variation in Poisson’s ratio as

\[ \nu = \nu_0 e^{\alpha_1 \frac{x}{a}}, \]  

where \( \nu_0 \) denotes Poisson’s ratio at reference temperature and \( \alpha_1 \) is non-homogeneity constant. Since the maximum value of Poisson’s ratio is less than 0.5, the numerical value of \( \alpha_1 \) (as it varies exponentially in this paper) can not be greater than 0.16 (approximately). Hence, the variation in Poisson’s ratio is taken from 0.0 to 0.15 (at most) in the calculation.

On using Eqs (11), (12), and (13) in Eq. (8), one obtains
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\[ D_1 = \left\{ \frac{E_0 \left[ 1 - \alpha \left( 1 - \frac{x^2}{a^2} \right) \left( 1 - \frac{y^2}{b^2} \right) \right]}{12 \left( 1 - \nu \right)^2} \right\} \left[ 1 + \beta \left( \frac{x^2}{a^2} \right)^3 \right]. \]  

(iv) The ends of the visco-elastic rectangular plate are supposed to be clamped. Due to the clamped boundary, the following conditions must be satisfied by deflection function \( \phi(x, y) \):

\[ \phi = \phi_x = 0, \quad x = 0, \ a \]
\[ \phi = \phi_y = 0, \quad y = 0, \ b \]  

Also, the corresponding two-term deflection function \( \phi(x, y) \) is taken as \[ \phi(x, y) = \phi_1(x, y)\phi_2(x, y), \]

where \( \phi_1(x, y) = \left[ \frac{x}{a} \left( 1 - \frac{x}{a} \right) \right]^2 \), and \( \phi_2(x, y) = \zeta_1 + \zeta_2 \left( 1 - \frac{x}{a} \right) \left( 1 - \frac{y}{b} \right) \).

where \( \zeta_1 \) and \( \zeta_2 \) are arbitrary constants.
Here \( \phi_1(x, y) \) satisfies all boundary conditions and \( \phi_2(x, y) \) provides two modes of vibration.

5. SOLUTION OF THE DIFFERENTIAL EQUATION OF MOTION

To obtain a frequency equation for the vibration of the rectangular plate, the authors used the Rayleigh–Ritz method. This method is based on the principle of conservation of energy, i.e. the maximum strain energy \( P \) must be equal to the maximum kinetic energy \( K \). So, it is necessary for the problem under consideration that \[ \delta(P - K) = 0, \]  

where

\[ K = (0.5)\rho \xi^2 \int_{0}^{a} \int_{0}^{b} h\phi^2 \, dy \, dx \]  

and

\[ P = (0.5)\int_{0}^{a} \int_{0}^{b} \left\{ \frac{\partial^2 \phi}{\partial x^2} \right\}^2 + \left\{ \frac{\partial^2 \phi}{\partial y^2} \right\}^2 + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left( \frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \, dy \, dx. \]  

To simplify and parameterize the present problem, non-dimensionalization is introduced as

\[ X = \frac{x}{a}, \quad Y = \frac{y}{a}. \]  

After using Eq. (20) in Eq. (18) and Eq. (19), one gets

\[ K^* = (0.5)\rho \xi^2 a^2 h_0 \int_{0}^{1} \int_{0}^{1} \frac{1}{h} (1 + \beta X^2) \phi^2 \, dY \, dX \]
and

\[ P^* = \frac{Q}{0 \to b/a} \left\{ \left[ 1 - \alpha (1 - X^2) \left( 1 - \frac{a^2}{b^2} Y^2 \right) \right] \left( 1 + \beta X^2 \right)^3 \right\} \left( 1 - 2\nu_0 e^{2\alpha X} \right) dYdX, \]

\[ \times \left\{ \frac{\partial^2 \phi}{\partial X^2} + \left( \frac{\partial^2 \phi}{\partial Y^2} \right)^2 + 2\nu_0 e^{\alpha X} \frac{\partial^2 \phi}{\partial X^2} \frac{\partial^2 \phi}{\partial Y^2} + 2 \left( 1 - \nu_0 e^{\alpha X} \right) \left( \frac{\partial^2 \phi}{\partial X \partial Y} \right)^2 \right\}. \]

(22)

where \( Q = \frac{E_0 h_0^3}{24a^2} \).

Using Eq. (21) and Eq. (22) in Eq. (17), one obtains

\[ (P^*_E - \lambda^2 K^*_E) = 0. \]

(23)

Here, \( \lambda^2 = \frac{12\rho a^4}{E_0 h_0^3} \) is the frequency parameter. Equation (23) contains two unknown constants, i.e. \( \zeta_1 \) and \( \zeta_2 \), arising due to the substitution of \( \phi(x, y) \). These two constants are to be determined as follows:

\[ \frac{\delta (P^*_E - \lambda^2 K^*_E)}{\delta \zeta_n} = 0, \quad n = 1, 2. \]

(24)

On simplifying Eq. (24), one gets

\[ C_{n1} \zeta_1 + C_{n2} \zeta_2 = 0, \quad n = 1, 2, \]

(25)

where \( C_{n1}, C_{n2} \) for \( n = 1, 2 \) involve structural parameters and the frequency parameter \( \lambda^2 \).

Equation (25) is a set of two simultaneous homogeneous equations with variables \( \zeta_1 \) and \( \zeta_2 \) having an infinite number of solutions. If one choose \( \zeta_1 = 1 \), one can easily evaluate \( \zeta_2 \), i.e.

\[ \zeta_2 = -\frac{C_{n1}}{C_{n2}}. \]

For a non-trivial solution, the determinant of the coefficients of Eq. (25) must be zero, i.e.

\[ \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0. \]

(26)

Equation (26) is a quadratic equation in \( \lambda^2 \) from which two values of \( \lambda^2 \) can be extracted.

The time period of the vibration of the visco-elastic plate is given by

\[ K = \frac{2\pi}{\lambda}. \]

(27)

6. SOLUTION FOR THE TIME FUNCTION \( \psi(t) \)

Time functions of free vibration of visco-elastic plates are defined by the general ordinary differential Eq. (7). Its form depends on the visco-elastic operator \( \tilde{D} \).
For Kelvin’s model, one gets [6]

$$\ddot{D} = 1 + \eta \frac{d}{G \, dt}.$$  
(28)

After using Eq. (28) in Eq. (7), Eq. (7) is modified as follows:

$$\ddot{\psi} + \xi^2 \left(1 + \eta \frac{d}{G \, dt}\right) \dot{\psi} = 0,$$

$$\ddot{\psi} + \xi^2 \frac{\eta}{G} \psi + \xi^2 \dot{\psi} = 0.$$  
(29)

Now, Eq. (29) is a differential equation of order two with respect to $t$ for the time function $\psi$. The solution of Eq. (29) can be obtained as [6]

$$\psi(t) = e^{\xi t} (C_1 \cosh \xi t + C_2 \sinh \xi t),$$  
(30)

where

$$a_1 = -\frac{\xi^2 \eta}{2G}, \quad b_1 = \xi \sqrt{1 - \left(\frac{\xi \eta}{2G}\right)^2},$$

and $C_1$ and $C_2$ are constants which can be determined easily from the initial conditions of the plate.

Let us take the initial conditions as

$$\psi = 1 \text{ at } t = 0$$
(31)

and

$$\dot{\psi} = 0 \text{ at } t = 0.$$  
(32)

Substituting Eq. (31) in Eq. (30), one gets

$$C_1 = 1.$$  
(33)

After using Eq. (32) in Eq. (30), one obtains

$$C_2 = \frac{-a_1}{b_1}.$$  
(34)

Substituting $C_1$ and $C_2$ from Eq. (33) and Eq. (34) in Eq. (30), Eq. (30) becomes

$$\psi(t) = e^{\xi t} \left[\cosh \xi t + \left(\frac{-a_1}{b_1}\right) \sinh \xi t\right].$$  
(35)

7. FORMULATION OF DEFLECTION

With the help of the values of $\zeta_1$ and $\zeta_2$, one can obtain deflection function $\phi$ as

$$\phi = \left[XY \frac{a}{b} (1-X) \left(1 - \frac{a}{b} Y\right)\right]^2 \times \left[1 + \left(\frac{-C_{11}}{C_{12}}\right) XY \frac{a}{b} (1-X) \left(1 - \frac{a}{b} Y\right)\right].$$  
(36)
On using Eq. (35) and Eq. (36) in Eq. (4), one gets

\[ w = \left[ \frac{XY a}{b} (1 - X) \left( 1 - \frac{a}{b} Y \right) \right]^2 \times \left[ 1 + \left( \frac{-C_{11}}{C_{12}} \right) \frac{XY a}{b} (1 - X) \left( 1 - \frac{a}{b} Y \right) \right] \times \left\{ e^{a t} \left[ \cos b_1 t + \left( \frac{a_1}{b_1} \right) \sin b_1 t \right] \right\}. \]

(37)

8. RESULTS AND DISCUSSION

In calculations, the following parameters are used for duralumin (an alloy of aluminium) [9]:

\[ E_0 = 7.08 \times 10^{10} \frac{N}{m^2}, \quad G = 2.632 \times 10^{10} \frac{N}{m^2}, \quad \eta = 14.612 \times 10^5 \frac{Ns}{m^2}, \quad \rho = 2.80 \times 10^3 \frac{kg}{m^3}, \]

\[ \nu_0 = 0.345, \quad h_0 = 0.01 m. \]

Table 1 presents the time period for the first two modes of vibration with a fixed aspect ratio \((a/b = 1.5)\) for different values of the non-homogeneity constant \(\alpha_i\) (i.e. \(\alpha_i = 0.0, 0.05, 0.10, 0.15\)) at the following combinations of the thermal gradient \(\alpha\) and the taper constant \(\beta\):

\[ \alpha = \beta = 0.0; \quad \alpha = \beta = 0.2; \quad \alpha = \beta = 0.6; \quad \alpha = \beta = 0.8. \]

It is interesting to note that the time period for the first two modes of vibration decreases with increasing \(\alpha_i\) for all combinations of \(\alpha\) and \(\beta\). As the combined value of \(\alpha\) and \(\beta\) increases from 0.0 to 0.8, the time period also decreases for both modes of vibration.

Table 2 tabulates the time period for the first two modes of vibration for fixed values of the thermal gradient and the taper constant, i.e. \(\alpha = \beta = 0.2\), for different values of the non-homogeneity constant \(\alpha_i\) and the aspect ratio \(a/b\). Here, the authors found a continuous decrement in the time period (for both modes) as \(\alpha_i\) increases from 0.0 to 0.15 and \(a/b\) increases simultaneously from 0.25 to 1.5.

<table>
<thead>
<tr>
<th>(a/b)</th>
<th>(\alpha_i = 0.0)</th>
<th>(\alpha_i = 0.05)</th>
<th>(\alpha_i = 0.10)</th>
<th>(\alpha_i = 0.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1705.27</td>
<td>416.69</td>
<td>1698.59</td>
<td>415.00</td>
</tr>
<tr>
<td>0.5</td>
<td>1585.19</td>
<td>395.12</td>
<td>1579.16</td>
<td>393.56</td>
</tr>
<tr>
<td>0.75</td>
<td>1367.51</td>
<td>347.39</td>
<td>1362.52</td>
<td>346.06</td>
</tr>
<tr>
<td>1.0</td>
<td>1100.30</td>
<td>281.61</td>
<td>1096.44</td>
<td>280.58</td>
</tr>
<tr>
<td>1.25</td>
<td>853.26</td>
<td>217.90</td>
<td>850.33</td>
<td>217.13</td>
</tr>
<tr>
<td>1.5</td>
<td>658.30</td>
<td>167.11</td>
<td>656.07</td>
<td>166.52</td>
</tr>
</tbody>
</table>
For $0 \leq X \leq 1$, $0 \leq Y \leq 1$ the deflection function $\phi(x, y)$ assumed in Eq. (16) shows the following properties:

\[
\phi(X, -\frac{a}{b}Y) = \phi(1 - X, -\frac{a}{b}Y),
\]
\[
\phi(X, -\frac{a}{b}Y) = \phi(X, 1 - \frac{a}{b}Y),
\]
\[
\phi(X, -\frac{a}{b}Y) = \phi(1 - X, 1 - \frac{a}{b}Y).
\]

Therefore, it has the same values for $X = 0.2$ and $X = 0.8$ as well as $X = 0.4$ and $X = 0.6$. The same is valid for $(a/b)Y$. Also for $X = 0.0$ and $X = 1.0$ or $Y = 0.0$ and $Y = 1.0$, deflection becomes zero. The deflection for both modes of vibration is reported at different values of $X$ and $Y$ for various values of plate parameters in Tables 3–5 as follows:

Table 3: $\alpha = \beta = 0.0$; $a/b = 1.5$; $\alpha_i = 0.0, 0.05, 0.10, 0.15$; $\psi = 0K, 5K$.
Table 4: $\alpha = \beta = 0.6$; $a/b = 1.5$; $\alpha_i = 0.0, 0.05, 0.10, 0.15$; $\psi = 0K, 5K$.
Table 5: $\alpha = \beta = 0.2$; $\alpha_i = 0.1$; $a/b = 0.25, 0.50, 0.75, 1.0, 1.25, 1.5$; $\psi = 0K, 5K$.

In Table 3, both modes of deflection (at $\psi = 0K$ and $\psi = 5K$) continuously decrease as the non-homogeneity constant $\alpha_i$ increases for each paired value of $X$ and $Y$. Also, it is interesting to note that the values of deflection for both modes of vibration are greater at $\psi = 0K$ as compared to $\psi = 5K$.

In Table 4, deflection shows different variations for $\psi = 0K$ and $\psi = 5K$. At $\psi = 0K$, deflection for both modes of vibration continuously increases as the non-homogeneity constant $\alpha_i$ increases for each paired value of $X$ and $Y$. At $\psi = 5K$, the first mode of deflection continuously decreases but the second mode of deflection continuously increases as the non-homogeneity constant $\alpha_i$ increases for each paired value of $X$ and $Y$.

Variation in deflection for both modes of vibration for various values of the aspect ratio can be explained with the help of Table 5. A rapid increment is found in deflection for both modes of vibration as the aspect ratio $a/b$ increases for each paired value of $X$ and $Y$ at $\psi = 0K$ and $\psi = 5K$.

Table 3. Deflection ($\times 10^{-5}$) vs. the non-homogeneity constant for $\alpha = \beta = 0.0$; $a/b = 1.5$ at $\psi = 0K$ and $\psi = 5K$ (in parentheses)

<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>$X$</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2</td>
<td>114.6210</td>
<td>39.5098</td>
<td>259.8370</td>
<td>6.3376</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(50.4673)</td>
<td>(1.8094)</td>
<td>(114.3570)</td>
<td>(0.2902)</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>114.6330</td>
<td>39.5099</td>
<td>259.8770</td>
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<td>(1.8036)</td>
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<td>(1.7939)</td>
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<td>(0.6847)</td>
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<td>(9.1098)</td>
<td>(0.6792)</td>
<td>(20.5647)</td>
<td>(1.2331)</td>
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Table 4. Deflection \((\times 10^{-5})\) vs the non-homogeneity constant for \(\alpha = \beta = 0.6; \ a/b = 1.5\) at \(\psi = 0K\) and \(\psi = 5K\) (in parentheses)

<table>
<thead>
<tr>
<th>(\alpha_i)</th>
<th>(X)</th>
<th>(Y)</th>
<th>(\psi = 0)</th>
<th>(\psi = 5)</th>
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<tr>
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<td>0.2</td>
<td>126.2400</td>
<td>39.5098</td>
<td>299.0520</td>
</tr>
<tr>
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<td>(53.6841)</td>
<td>(1.7888)</td>
<td>(127.1730)</td>
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<td>0.2</td>
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<td>(1.7898)</td>
<td>(126.9420)</td>
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<td>(1.7921)</td>
<td>(126.6990)</td>
</tr>
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<td>126.5740</td>
<td>39.6865</td>
<td>300.1800</td>
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<tr>
<td></td>
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<td>(53.3206)</td>
<td>(1.7958)</td>
<td>(126.4540)</td>
</tr>
<tr>
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<td>0.6</td>
<td>21.7864</td>
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<td>50.2011</td>
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<td>(6.6750)</td>
<td>(21.3482)</td>
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<td>0.6</td>
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<td>14.9729</td>
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<td>(6.753)</td>
<td>(21.2971)</td>
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<td>(6.7671)</td>
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<td>0.6</td>
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<tr>
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<td></td>
<td>(9.1888)</td>
<td>(6.7755)</td>
<td>(21.1851)</td>
</tr>
</tbody>
</table>

Table 5. Deflection \((\times 10^{-5})\) vs the aspect ratio for \(\alpha = \beta = 0.2; \ \alpha_i = 0.1\) at \(\psi = 0K\) and \(\psi = 5K\) (in parentheses)

<table>
<thead>
<tr>
<th>(a/b)</th>
<th>(X)</th>
<th>(Y)</th>
<th>(\psi = 0)</th>
<th>(\psi = 5)</th>
</tr>
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<td>(3.3069)</td>
<td>(9.8662)</td>
</tr>
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<td>(15.2833)</td>
<td>(3.6924)</td>
<td>(35.1165)</td>
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<td>0.2</td>
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<td></td>
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<td>(5.1270)</td>
<td>(65.8702)</td>
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<tr>
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<td>(40.7444)</td>
<td>(4.6322)</td>
<td>(92.8699)</td>
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<td>(2.9799)</td>
<td>(110.6060)</td>
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<td>(50.7121)</td>
<td>(1.7872)</td>
<td>(116.2440)</td>
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9. COMPARISON AND CONCLUSIONS

The frequency for both modes of vibration in the present paper is compared with the frequency in [8] at the corresponding values of structural parameters for two cases:

Case (i) \(\beta = 0.0; \ \alpha_i = 0.1; \ \alpha = 0.0, 0.2, 0.4, 0.6, 0.8\).

Case (ii) \(\alpha = 0.0; \ \alpha_i = 0.1; \ \beta = 0.0, 0.2, 0.4, 0.6, 0.8\).

A graphical representation is provided for both the above cases (Fig. 2, Graph I for case (i) and Graph II for case (ii)). Graph I clearly shows that the frequency in the present paper is less than the frequency in [8] for both modes of vibration. But in Graph II, both modes of frequency in the present paper are greater than in [8].

The frequencies in the present paper and in [8] coincide only at \(\alpha = \alpha_i = \beta = 0.0\).

Based on the above graphical comparison, the authors conclude the following:

(i) In case of bi-parabolic variation in temperature, low frequency vibrations comparable to [8] can be obtained.

(ii) Tapering directly affects the vibrational behaviour of structures or plates. Frequency is higher in case of parabolic tapering (present paper) as compared to exponential tapering [8].
(iii) Frequency can be controlled actively by using appropriate values of structural parameters.

(iv) The present study confirms previous findings and also provides futuristic numerical data for researchers and design engineers to enhance the efficiency and reliability of machines and mechanical structures.

Fig. 2. Comparison of the frequencies of the two modes in the present study and in [8]. Graph I: $\beta = 0.0$; Graph II: $\alpha = 0.0$. 

Graph-I

Graph-II
REFERENCES


Struktuursete parameetrite mõju ristkülikulise mittehomogeense viskoelastse plaadi vibratsioonidele

Anupam Khanna ja Narinder Kaur