



## Remarks on (super-)accelerating cosmological models in general scalar–tensor gravity

Laur Järv<sup>a\*</sup>, Piret Kuusk<sup>a</sup>, and Margus Saal<sup>b</sup>

<sup>a</sup> Institute of Physics, University of Tartu, Riia 142, 51014 Tartu, Estonia

<sup>b</sup> Tartu Observatory, 61602 Tõravere, Estonia

Received 3 June 2009, accepted 16 July 2010

**Abstract.** We consider Friedmann–Lemaître–Robertson–Walker cosmological models in the framework of general scalar–tensor theories of gravity (STG) with arbitrary coupling functions, set in the Jordan frame. First we describe the general properties of the phase space in the case of barotropic matter fluid and scalar field potential for any spatial curvature (flat, spherical, hyperbolic). Then we address the question under which conditions epochs of accelerated and super-accelerated expansion are possible in STG. For flat models filled with dust matter (and vanishing potential) we give a necessary condition on the coupling function of the scalar field which must be satisfied to allow acceleration and super-acceleration. This is illustrated by a specific example.

**Key words:** scalar–tensor gravity, super-accelerating cosmological models.

### 1. INTRODUCTION

The last decade has produced an abundance of cosmological precision data, leading to surprising results and implications. Approaching the statistics of observations by more relaxed priors suggests that the expansion of the Universe as measured by the scale factor  $a$  is not only accelerating ( $\frac{\ddot{a}}{a} > 0$ ), but might be also about to enter into a super-accelerating phase ( $\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} > 0$ ), sometimes dubbed as “crossing the phantom divide” [1–5]. This possibility cannot be accommodated in the cosmological Concordance Model based on the Einstein equations with a cosmological constant in the framework of general relativity (GR). If one prefers to play within the traditional GR, then the onset of super-acceleration can be invoked by adding another matter component with unusual “phantom” properties [6–8]. An alternative explanation would require superseding GR by a more general theory of gravity. Examples of super-accelerating dust matter solutions have been studied, for instance, in the context of  $f(R)$  [9,10] and scalar–tensor [11–16] theories.

Scalar–tensor theories of gravity (STG) employ a scalar field  $\Psi$  besides the usual space-time metric tensor  $g_{\mu\nu}$  to describe the gravitational interaction. Scalar field is in the role of a variable gravitational “constant”, leaving tensorial metric field and its geodesics to act as trajectories of freely falling particles as in GR. In general, STG form a collection of theories which contain two functional degrees of freedom, a coupling function  $\omega(\Psi)$ , and a scalar potential  $V(\Psi)$ . Each distinct functional form of these two functions gives us a distinct theory of gravitation together with its field equations. It is of considerable interest to determine which members of this family of theories allow solutions (model Universes) exhibiting periods of accelerating and super-accelerating expansion without introducing any unusual matter component.

The study of global properties of solutions can be greatly facilitated by the mathematical methods of dynamic systems and phase space. Several previous detailed studies which have considered STG cosmology as a dynamic system have focused upon examples with specific coupling functions [17–23]. The main properties of the corresponding general phase space geometry were outlined by Faraoni [24] and us [25].

\* Corresponding author, [laur.jarv@ut.ee](mailto:laur.jarv@ut.ee)

The plan of the paper is the following. Section 2 introduces STG field equations for homogeneous and isotropic cosmological models. In Section 3 we describe the phase space in the most general case: one barotropic matter fluid component, non-vanishing scalar field potential, and arbitrary spatial geometry (flat, spherical, hyperbolic), thus generalizing the results of previous studies [24,25]. In Section 4 we investigate the conditions under which accelerated and super-accelerated expansion is possible, and also when the solutions enter or leave the epoch of accelerated and super-accelerated expansion. In the simplest and phenomenologically most relevant case of dust matter, vanishing potential, and flat spatial geometry ( $k = 0$ ) we give a necessary condition on the coupling function  $\omega(\Psi)$  which must be satisfied for acceleration and super-acceleration to be possible at all. These considerations are illustrated by an example of a particular STG where some solutions undergo a phase of super-acceleration while others do not.

## 2. THE EQUATIONS OF SCALAR-TENSOR COSMOLOGY

We consider a general scalar-tensor theory in the Jordan frame given by the action functional

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \Psi R(g) - \frac{\omega(\Psi)}{\Psi} \nabla^\rho \Psi \nabla_\rho \Psi - 2\kappa^2 V(\Psi) \right] + S_m(g_{\mu\nu}, \chi_m). \quad (1)$$

Here  $\omega(\Psi)$  is a coupling function and  $V(\Psi)$  is a scalar potential,  $\nabla_\mu$  denotes the covariant derivative with respect to the metric  $g_{\mu\nu}$ ,  $\kappa^2$  is the non-variable part of the gravitational constant, and  $S_m$  is the matter contribution to the action as all other fields are included in  $\chi_m$ . In order to keep the effective gravitational constant  $\frac{\kappa^2}{\Psi}$  positive, we assume that  $0 < \Psi < \infty$ .

The field equations for the Friedmann-Lemaître-Robertson-Walker (FLRW) line element

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right) \quad (2)$$

with curvature parameter  $k = 0$  (flat),  $+1$  (spherical),  $-1$  (hyperbolic), and perfect barotropic fluid matter,  $p = w\rho$ , read

$$H^2 = -H \frac{\dot{\Psi}}{\Psi} + \frac{1}{6} \frac{\dot{\Psi}^2}{\Psi^2} \omega(\Psi) + \frac{\kappa^2 \rho}{3 \Psi} + \frac{\kappa^2 V(\Psi)}{3 \Psi} - K, \quad (3)$$

$$2\dot{H} + 3H^2 = -2H \frac{\dot{\Psi}}{\Psi} - \frac{1}{2} \frac{\dot{\Psi}^2}{\Psi^2} \omega(\Psi) - \frac{\ddot{\Psi}}{\Psi} - \frac{\kappa^2}{\Psi} w\rho + \frac{\kappa^2}{\Psi} V(\Psi) - K, \quad (4)$$

$$\begin{aligned} \ddot{\Psi} = & -3H\dot{\Psi} - \frac{1}{2\omega(\Psi)+3} \frac{d\omega(\Psi)}{d\Psi} \dot{\Psi}^2 + \frac{\kappa^2}{2\omega(\Psi)+3} (1-3w) \rho \\ & + \frac{2\kappa^2}{2\omega(\Psi)+3} \left[ 2V(\Psi) - \Psi \frac{dV(\Psi)}{d\Psi} \right], \end{aligned} \quad (5)$$

where  $H \equiv \dot{a}/a$ ,  $K = \frac{k}{a^2}$ . The matter conservation law is the usual

$$\dot{\rho} + 3H(w+1)\rho = 0 \quad (6)$$

and it is reasonable to assume positive matter energy density,  $\rho \geq 0$ .

## 3. PHASE SPACE

The system (3)–(6) is characterized by five variables  $\{\Psi, \dot{\Psi}, H, a, \rho\}$ , but one of them is algebraically related to the others via the Friedmann equation (3). Since the scale factor  $a$  is not physically observable, it is reasonable

to eliminate  $K$  by Eq. (3). This leads to a phase space spanned by four variables  $\{\Psi, \dot{\Psi}, H, \rho\}$ . By defining  $\Psi \equiv x, \dot{\Psi} \equiv y$  the dynamic system corresponding to equations (3)–(6) can be written as follows:

$$\dot{x} = y, \tag{7}$$

$$\dot{y} = -\frac{1}{2\omega(x)+3} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2 (1-3w)\rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] - 3Hy, \tag{8}$$

$$\begin{aligned} \dot{H} = & \frac{1}{2x(2\omega(x)+3)} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2 (1-3w)\rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] \\ & - H^2 + H \frac{y}{x} - \omega(x) \frac{y^2}{3x^2} - \frac{\kappa^2}{6x} (1+3w)\rho + \frac{\kappa^2 V(x)}{3x}, \end{aligned} \tag{9}$$

$$\dot{\rho} = -3H(1+w)\rho. \tag{10}$$

The phase space may be imagined as a four-dimensional box filled by the spaghetti of one-dimensional trajectories (orbits of solutions) which do not intersect with each other except for special points known as fixed (critical, equilibrium) points. As the curvature invariants of FLRW metric are proportional to  $H$  and  $\dot{H}$ , the phase space boundaries  $|H| \rightarrow \infty, |\rho| \rightarrow \infty,$  and  $|\dot{\Psi}| \rightarrow \infty$  generically entail a space-time singularity. Analogously, the limit  $\Psi \rightarrow 0$  in general implies diverging  $|H|$  or  $|\dot{H}|$  and poses a space-time singularity, obstructing the solutions from safely passing from positive to negative values of  $\Psi$  (from “attractive” to “repulsive” gravity). The limit  $\Psi \rightarrow \infty$  does not call forth a space-time singularity; however, the gravitational “constant” vanishes.

Within this box there could also be singular hypersurfaces perpendicular to the  $\Psi$  axis, depending on the form of  $\omega$  and  $V$ . So, in general terms the limit  $V \rightarrow \infty$  renders the system singular, and also  $2\omega + 3 \rightarrow 0$  implies  $|\dot{H}| \rightarrow \infty$  with the same conclusion that passing through  $\omega(\Psi) = -\frac{3}{2}$  (corresponding to the change of the sign of the scalar field kinetic term in the Einstein frame action) would entail a space-time singularity and is impossible. The limit  $\frac{1}{2\omega+3} \rightarrow 0$  is also marred by a singularity, unless simultaneously

$$\dot{\Psi} \rightarrow 0, \quad \omega \dot{\Psi}^2 \rightarrow 0, \quad \frac{1}{(2\omega+3)^2} \frac{d\omega}{d\Psi} \rightarrow \text{finite}. \tag{11}$$

The latter situation is particularly interesting, since in this limit the system coincides with the FLRW equations of general relativity [25].

The trajectories corresponding to the flat FLRW geometry ( $k = 0$ ) lie on the 3-surface

$$\mathcal{F} : F(x, y, H, \rho) \equiv H^2 + H \frac{y}{x} - \frac{y^2}{6x^2} \omega(x) - \frac{\kappa^2 \rho}{3x} - \frac{\kappa^2 V(x)}{3x} = 0 \tag{12}$$

due to the constraint (3). The trajectories corresponding to spherical and hyperbolic models remain on either side of this surface. In principle the geometry of the 3-surface  $\mathcal{F}$  in the four-dimensional phase space is rather complicated to visualize, but a few general characteristics can still be given. We may write Eq. (12) as

$$\frac{\left(H + \frac{y}{2x}\right)^2}{\frac{\kappa^2(\rho+V)}{3x}} - \frac{y^2}{\frac{4\kappa^2x(\rho+V)}{2\omega+3}} = 1, \tag{13}$$

which for fixed  $\rho$  and  $x$  can be recognized as describing familiar conic sections: (1) for  $\rho + V > 0, 2\omega + 3 > 0$  a hyperbola on the  $(H + \frac{y}{2x}, y)$  plane, (2) for  $\rho + V > 0, 2\omega + 3 < 0$  an ellipse also on the  $(H + \frac{y}{2x}, y)$  plane, while (3) for  $\rho + V < 0, 2\omega + 3 > 0$  a hyperbola on the  $(y, H + \frac{y}{2x})$  plane. The case (4)  $\rho + V < 0, 2\omega + 3 < 0$  is not realized as real solutions are absent. This result establishes that the intersection of the 3-surface  $\mathcal{F}$  with the (fixed  $\rho$ , fixed  $x$ ) 2-plane is constituted in either one piece (ellipse) or two pieces (hyperbola). Thus in case (1) the allowed phase space is divided into two separate regions, the “upper” region where  $H + \frac{y}{2x} > 0$  and the “lower” region where  $H + \frac{y}{2x} < 0$ , and there is no way the trajectories can travel from one region to the other. In case (2) these two regions meet along a 2-surface where  $H + \frac{y}{2x} = 0$ , and the trajectories can in principle cross from one region to the other. In case (3) there are again two separate parts, now characterized by  $y > 0$  and  $y < 0$ , respectively. At first it may be difficult to find a direct physical interpretation for the quantity  $H + \frac{y}{2x}$  that characterizes the “upper” and “lower” region in cases (1) and (2), but it turns out that this combination is equal to the Hubble parameter in the Einstein frame [25,26], and thus the “upper” region corresponds to universes which expand in the Einstein frame, while the “lower” region has universes which contract in the Einstein frame.

**Table 1.** For certain  $k$  the Friedmann equation constrains the values of  $\Psi \equiv y$  (14) and  $H$  (15)

		Allowed range of $\Psi$ for $k = 0, +1$	Allowed range of $H$ for $k = 0, -1$	
(1a)	$\rho + V \geq 0$	$0 \leq \omega \leq \infty$	$0 \leq \Psi^2 \leq \infty$	$\frac{2\kappa^2\omega(\rho+V)}{3\Psi(2\omega+3)} \leq H^2 \leq \infty$
(1b)	$\rho + V \geq 0$	$-\frac{3}{2} \leq \omega \leq 0$	$0 \leq \Psi^2 \leq \infty$	$0 \leq H^2 \leq \infty$
(2)	$\rho + V > 0$	$-\infty \leq \omega \leq -\frac{3}{2}$	$0 \leq \Psi^2 \leq \frac{4\kappa^2(\rho+V)\Psi}{ 2\omega+3 }$	$0 \leq H^2 \leq \frac{2\kappa^2\omega(\rho+V)}{3\Psi(2\omega+3)}$
		Allowed range of $\Psi$ for $k = 0, +1$	Allowed range of $H$ for $k = 0, +1$	
(3a)	$\rho + V \leq 0$	$0 \leq \omega \leq \infty$	$\frac{4\kappa^2 \rho+V \Psi}{2\omega+3} \leq \Psi^2 \leq \infty$	$0 \leq H^2 \leq \infty$
(3b)	$\rho + V \leq 0$	$-\frac{3}{2} \leq \omega \leq 0$	$\frac{4\kappa^2 \rho+V \Psi}{2\omega+3} \leq \Psi^2 \leq \infty$	$\frac{2\kappa^2\omega(\rho+V)}{3\Psi(2\omega+3)} \leq H^2 \leq \infty$
(4)	$\rho + V < 0$	$-\infty < \omega < -\frac{3}{2}$	-	-

Related information can be also established by another approach. With general  $k$  we may solve the Friedmann constraint, Eq. (3), for  $H$  and then the condition for all variables to be real valued imposes an inequality

$$(2\omega(x) + 3) \frac{y^2}{12x^2} + \frac{\kappa^2(\rho + V(x))}{3x} \geq K. \tag{14}$$

In terms of physics this inequality can be interpreted as a restriction on the allowed values of  $y$ . Table 1 summarizes the situation. By the allowed range for a given value of  $k$  we mean that if  $y$  satisfies the inequality listed, then it is possible to find a real-valued  $a$  which fits the Friedmann equation. Thus for  $k = 0$  and  $k = +1$  there is no restriction in case (1), while case (4) is completely ruled out since no real solutions compatible with the Friedmann constraint exist. For  $k = -1$  there are no restrictions.

Similarly, solving the Friedmann constraint for  $y$  leads to another inequality,

$$(2\omega(x) + 3)H^2 - \frac{2\kappa^2\omega(x)}{3x}(\rho + V(x)) \geq -2K\omega(x), \tag{15}$$

which can be interpreted as a restriction on the allowed values of  $H$  (given also in Table 1). Analogously, once  $\omega(x)$  and  $V(x)$  are specified, we may get a third inequality from solving the Friedmann constraint for  $x$  as well.

#### 4. ACCELERATION AND SUPER-ACCELERATION

In the four-dimensional phase space there could be regions where the trajectories exhibit super-accelerating behaviour, marked by the condition

$$S(x, y, H, \rho) \equiv \frac{1}{2x(2\omega(x) + 3)} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2(1 - 3\omega)\rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] - H^2 + H \frac{y}{x} - \omega(x) \frac{y^2}{3x^2} - \frac{\kappa^2}{6x} (1 + 3\omega)\rho + \frac{\kappa^2 V(x)}{3x} > 0, \tag{16}$$

and surrounded by regions of accelerated expansion, delineated by

$$A(x, y, H, \rho) \equiv \frac{1}{2x(2\omega(x) + 3)} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2(1 - 3\omega)\rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] + H \frac{y}{x} - \omega(x) \frac{y^2}{3x^2} - \frac{\kappa^2}{6x} (1 + 3\omega)\rho + \frac{\kappa^2 V(x)}{3x} > 0. \tag{17}$$

For a cursory comparison with general relativity let us recall that in GR super-acceleration requires matter with barotropic index  $w < -1$ , while acceleration demands  $w < -\frac{1}{3}$ . Cosmological constant behaves as a barotropic fluid with  $w = -1$ . Equations (16), (17) readily reveal that identical conditions are recovered in STG at the GR limit (11), where the fixed value of the potential is read as the cosmological constant, and the Friedmann constraint (3) should be taken into account along with (16).

However, away from the GR limit, new possibilities occur. First, irrespective of the matter content, the scalar field itself may trigger accelerated and super-accelerated expansion in the domain where  $\omega < 0$  or  $\frac{1}{2\kappa(2\omega+3)} \frac{d\omega}{d\Psi} > 0$ . Second, acceleration and super-acceleration may also occur for matter with  $w > \frac{1}{3}$  in the domain where  $2\omega + 3 > 0$ , or for  $w < \frac{1}{3}$  in the domain where  $2\omega + 3 < 0$ . Third, the overall effect of the potential is considerably more complicated than that of a simple cosmological constant, depending on the derivative  $\frac{dV}{d\Psi}$  as well as the sign of  $2\omega + 3$ , and even implying a possibility that a constant negative potential may lead to super-acceleration, provided that  $2\omega + 3 > 0$ .

The region of super-acceleration is bounded by the 3-surface  $\mathcal{S} : S(x, y, H, \rho) = 0$ . The circumstance whether the trajectories enter this region can be read off from the scalar product of the gradient normal to  $\mathcal{S}$  and the tangent vector of the phase flow  $T^i = (\dot{x}, \dot{y}, \dot{H}, \dot{\rho})$ , namely  $\nabla_i S \cdot T^i \Big|_{\mathcal{S}} > 0$ . A completely analogous condition arises for the surface  $\mathcal{A} = 0$  bounding the region of acceleration.

These results, expressed in full generality, hint ample possibilities for acceleration and super-acceleration, but in order to come up with more exact conditions one has to narrow down the scope a bit. Therefore let us focus upon the physically most interesting case of spatially flat ( $k = 0$ ) universe filled with dust matter ( $w = 0$ ) and vanishing potential. In this case Eq. (4) can be written as

$$\begin{aligned} \dot{H} &= -\frac{H^2}{2} - \frac{5}{12} \frac{\omega \dot{\Psi}^2}{\Psi^2} - \frac{\ddot{\Psi}}{2\Psi} - \frac{\kappa^2 \rho}{3\Psi} \\ &= -2H^2 + \frac{\kappa^2 \omega \rho}{3\Psi(2\omega+3)} + \frac{\dot{\Psi}^2}{2\Psi^2} \left( \frac{\Psi}{2\omega+3} \frac{d\omega}{d\Psi} - \frac{\omega}{3} \right). \end{aligned} \quad (18)$$

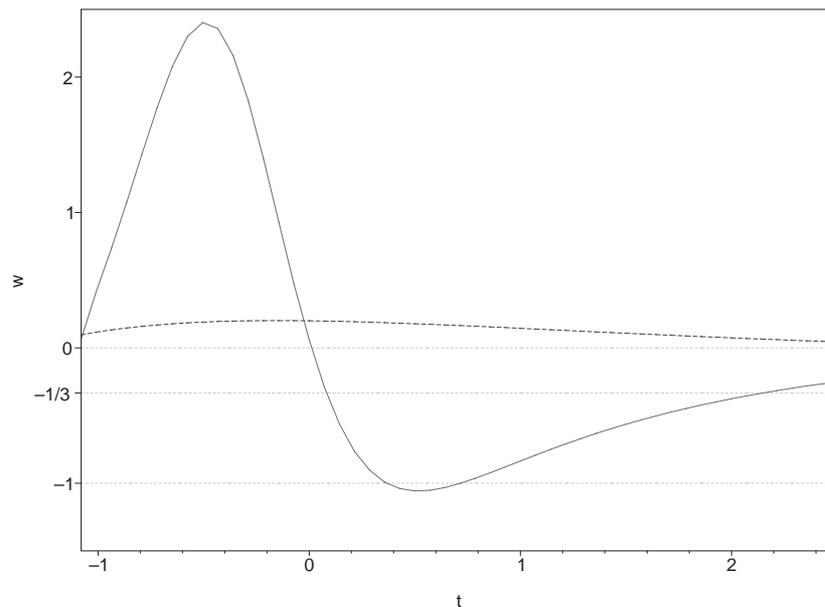
Here the first line informs that super-acceleration is only possible if  $\omega < 0$ , or  $\ddot{\Psi} < 0$ . From the second line, which has taken Eq. (5) into account, we can read off a necessary condition on the form of the coupling function  $\omega$  for super-acceleration to be possible

$$\mathcal{C} = \frac{\Psi}{2\omega+3} \frac{d\omega}{d\Psi} - \frac{\omega}{3} > 0, \quad (19)$$

assuming  $2\omega + 3 > 0$ . The reason is that if  $\omega > 0$ , the Friedmann constraint imposes  $\frac{\kappa^2 \omega(\rho)}{3\Psi(2\omega+3)} \leq \frac{H^2}{2}$ , and the only positive contribution towards super-acceleration can arise from the third term in (18). The same is true for  $-\frac{3}{2} < \omega < 0$  since then the second term is negative itself. (It is easier to achieve super-acceleration if  $2\omega + 3 < 0$ , but this option is not so lucrative since in the Einstein frame, where the tensor and scalar degrees are not mixed, the kinetic energy of the latter is negative, and thus problematic [27].)

Note that Eq. (19) provides only a necessary but not sufficient condition for super-accelerating solutions to be present in a model. More exactly, it states that in the domain of  $\Psi$ , where (19) holds, there may be solutions which undergo super-accelerated expansion. In the domain of  $\Psi$ , where (19) does not hold, super-acceleration is not possible. Therefore, given a zoo of all possible forms of  $\omega$ , it can be used to filter out and discard from further investigation those forms of  $\omega$  that are decidedly infertile with respect to super-acceleration.

Finally, as an illustration, let us consider  $2\omega + 3 = \frac{1}{2(1-\Psi)}$ . Here  $2\omega + 3 > 0$  and (19) holds in the domain  $0 < \Psi < 1$ . Inspection of the phase space flow reveals that the trajectories on the “upper sheet” ( $H + \frac{\dot{\Psi}}{2\Psi} > 0$ ), where most of the expanding,  $H > 0$ , models lie, belong to two typical classes: either exhibiting a super-accelerating phase or not, see Fig. 1. The dynamics has been characterized by the evolution of the effective barotropic index,  $w_{\text{eff}} = -1 - \frac{2\dot{H}}{3H^2}$ , defined as an analogy to single component barotropic fluid FLRW models in GR. In particular,  $w_{\text{eff}} = 0$  characterizes the decelerating evolution of usual dust matter,  $w_{\text{eff}} < -\frac{1}{3}$  is required for acceleration, while  $w_{\text{eff}} = -1$  corresponds to the “phantom divide line” below which super-acceleration occurs.



**Fig. 1.** Two typical cosmological evolutions in the case of  $2\omega + 3 = \frac{1}{2(1-\Psi)}$ ,  $V(\Psi) \equiv 0$ ,  $w = 0$ ,  $k = 0$ : one solution (solid line) goes through a brief period of super-accelerated expansion (where  $w_{\text{eff}} < -1$ ), the other solution (dashed line) does not.

## ACKNOWLEDGEMENTS

This work was supported by the Estonian Science Foundation (grant No. 7185) and by the Estonian Ministry for Education and Research (support grant No. SF0180013s07). MS also acknowledges the Estonian Science Foundation Post-doctoral research grant No. JD131. The authors would like to thank the organizers of the Baltic–Nordic AGMF Workshop '08, where part of the present paper was presented.

## REFERENCES

1. Sahni, V. and Starobinsky, A. Reconstructing dark energy. *Int. J. Mod. Phys. D*, 2006, **15**, 2105–2132.
2. Alam, U., Sahni, V., and Starobinsky, A. A. Exploring the properties of dark energy using type Ia supernovae and other datasets. *JCAP*, 2007, **0702**, 011.
3. Nesseris, S. and Perivolaropoulos, L. Crossing the phantom divide: theoretical implications and observational status. *JCAP*, 2007, **0701**, 018.
4. Wu, P. and Yu, H. W. Probing the cosmic acceleration history and the properties of dark energy from the ESSENCE supernova data with a model independent method. *JCAP*, 2008, **0802**, 019.
5. Perivolaropoulos, L. and Shafieloo, A. Bright high  $z$  SNIa: a challenge for  $\Lambda$ CDM? arXiv:0811.2802 [astro-ph].
6. Caldwell, R. R. A phantom menace? *Phys. Lett. B*, 2002, **545**, 23–29.
7. Vikman, A. Can dark energy evolve to the phantom? *Phys. Rev. D*, 2005, **71**, 023515.
8. Caldwell, R. R. and Doran, M. Dark-energy evolution across the cosmological-constant boundary. *Phys. Rev. D*, 2005, **72**, 043527.
9. Amendola, L. and Tsujikawa, S. Phantom crossing, equation-of-state singularities, and local gravity constraints in  $f(R)$  models. *Phys. Lett. B*, 2008, **660**, 125–132.
10. Bamba, K., Geng, C. Q., Nojiri, S., and Odintsov, S. D. Crossing of the phantom divide in modified gravity. arXiv:0810.4296 [hep-th].
11. Faraoni, V. Superquintessence. *Int. J. Mod. Phys. D*, 2002, **11**, 471–482.
12. Elizalde, E., Nojiri, S., and Odintsov, S. D. Late-time cosmology in (phantom) scalar–tensor theory: dark energy and the cosmic speed-up. *Phys. Rev. D*, 2004, **70**, 043539.
13. Perivolaropoulos, L. Crossing the phantom divide barrier with scalar tensor theories. *JCAP*, 2005, **0510**, 001.
14. Demianski, M., Piedipalumbo, E., Rubano, C., and Tortora, C. Accelerating universe in scalar tensor models: confrontation of theoretical predictions with observations. *Astron. Astrophys.*, 2006, **454**, 55–66.
15. Gannouji, R., Polarski, D., Ranquet, A., and Starobinsky, A. A. Scalar–tensor models of normal and phantom dark energy. *JCAP*, 2006, **0609**, 016.
16. Tsujikawa, S., Uddin, K., Mizuno, S., Tavakol, R., and Yokoyama, J. Constraints on scalar–tensor models of dark energy from observational and local gravity tests. *Phys. Rev. D*, 2008, **77**, 103009.
17. Billyard, A., Coley, A., and Ibáñez, J. On the asymptotic behaviour of cosmological models in scalar–tensor theories of gravity. *Phys. Rev. D*, 1998, **59**, 023507.

18. Gunzig, E., Faraoni, V., Figueiredo, A., Rocha, T. M., and Brenig, L. The dynamical system approach to scalar field cosmology. *Class. Quant. Grav.*, 2000, **17**, 1783–1814.
19. Gunzig, E., Saa, A., Brenig, L., Faraoni, V., Rocha Filho, T. M., and Figueiredo, A. Superinflation, quintessence, and nonsingular cosmologies. *Phys. Rev. D*, 2001, **63**, 067301.
20. Carvalho, F. C. and Saa, A. Non-minimal coupling, exponential potentials and the  $w < -1$  regime of dark energy. *Phys. Rev. D*, 2004, **70**, 087302.
21. Järv, L., Kuusk, P., and Saal, M. The dynamics of scalar–tensor cosmology from RS two-brane model. *Phys. Rev. D*, 2007, **75**, 023505.
22. Järv, L., Kuusk, P., and Saal, M. Scalar–tensor cosmology at the general relativity limit: Jordan vs Einstein frame. *Phys. Rev. D*, 2007, **76**, 103506.
23. Hrycyna, O. and Szydlowski, M. Extended quintessence with non-minimally coupled phantom scalar field. *Phys. Rev. D*, 2007, **76**, 123510.
24. Faraoni, V. Phase space geometry in scalar–tensor cosmology. *Ann. Phys. (N.Y.)*, 2005, **317**, 366–382.
25. Järv, L., Kuusk, P., and Saal, M. Scalar–tensor cosmologies: fixed points of the Jordan frame scalar field. *Phys. Rev. D*, 2008, **78**, 083530.
26. Faraoni, V., Jensen, M. N., and Theuerkauf, S. A. Non-chaotic dynamics in general-relativistic and scalar–tensor cosmology. *Class. Quant. Grav.*, 2006, **23**, 4215–4230.
27. Esposito-Farese, G. and Polarski, D. Scalar–tensor gravity in an accelerating universe. *Phys. Rev. D*, 2001, **63**, 063504.

### **Mõned märkused superkiirenevate kosmoloogiliste mudelite kohta gravitatsiooni üldises skalaar-tensorteoorias**

Laur Järv, Piret Kuusk ja Margus Saal

On vaadeldud Friedmanni-Lemaître'i-Robertsoni-Walkeri kosmoloogilisi mudeleid Jordani raamis esitatud gravitatsiooni üldises skalaar-tensorteoorias (STG), mis sisaldab suvalist seosefunktsiooni. On kirjeldatud faasiruumi üldisi omadusi barotroopse vedeliku tüüpi aine ja skalaarvälja potentsiaali olemasolu korral ruumilise kõveruse kõigil juhtudel (tasane, sfääriline, hüperboolne). On uuritud küsimust, millistel tingimustel saab neis mudelites universumi paisumine olla kiirenev või superkiirenev. Selliste tasaste mudelite jaoks, kus kosmoloogiliseks aineks on tolmu ja skalaarvälja potentsiaal puudub, on antud tarvilik tingimus, mida seosefunktsioon peab rahuldama, selleks et universum võiks paisuda kiirenevalt või superkiirenevalt. Arutlused on illustreeritud sobiva konkreetse näitega.