Proceedings of the Estonian Academy of Sciences,
2010, **59**, 4, 265–271
doi: 10.3176/proc.2010.4.03
Available online at www.eap.ee/proceedings

MATHEMATICS

Computing the index of Lie algebras

Hadjer Adimi[a] and Abdenacer Makhlouf[b*]

[a] Centre universitaire de Bordj Bou Arreridj, Département de Mathématiques, Bordj Bou Arreridj, Algeria
[b] Université de Haute Alsace, Laboratoire de Mathématiques, Informatique et Applications, 4, rue des Frères Lumière F-68093 Mulhouse, France

Received 17 April 2009, accepted 21 October 2009

Abstract. The aim of this paper is to compute and discuss the index of Lie algebras. We consider the n-dimensional Lie algebras for $n < 5$ and the case of filiform Lie algebras which form a special class of nilpotent Lie algebras. We compute the index of generalized Heisenberg algebras and graded filiform Lie algebras L_n and Q_n. We also discuss the evolution of the Lie algebra index by deformation.

Key words: Lie algebra, index, regular vector, deformation.

1. INTRODUCTION

The index theory of Lie algebras was intensively studied by Elashvili (see [5–8]), in particular the case of semi-simple Lie algebras and Frobenius Lie algebras. He classified all the algebraic Frobenius algebras up to dimension 6. In [3], the authors connect the computation of the index to combinatorial theory of meanders and evaluate the index of a Lie algebra of seaweed type, which is equal to the number of cycles in an associated permutation. The index of semi-simple Lie algebras was also studied in [21]. The authors of that paper consider a semi-simple Lie algebra $\mathscr{G}$ with a Cartan subalgebra h, R its corresponding root system, π a base of R, and S, T subsets of π. They provide an upper bound for the index of $\mathscr{G}_{S,T}$, the direct sum of h, and the sum of the root spaces for the positive roots in the space spanned by S and the sum of the root spaces for the negative roots in the space spanned by T. They then verify that this inequality is actually an equality in a number of special cases and conjecture that equality holds in all cases. See also [20], where the index of a Borel subalgebra of a semi-simple Lie algebra is determined.

The aim of this paper is to compute the index of Lie algebras in low dimensions and in general for some special cases. In Section 2 we summarize the index theory of Lie algebras. Then, in Section 3, we recall the classification of n-dimensional Lie algebras for $n < 5$ and compute the indexes for all these Lie algebras. Section 4 is dedicated to nilpotent Lie algebras and specially to filiform Lie algebras. We consider the generalized Heisenberg Lie algebras and the two graded filiform Lie algebras L_n and Q_n. Notice that L_n plays an important role in the study of filiform and nilpotent Lie algebras. It is known that any n-dimensional filiform Lie algebra may be obtained by deformation of the one of the filiform Lie algebras L_n. In the last Section we study the evolution by deformation of the index of a Lie algebra. We prove that the index of a Lie algebra decreases by deformation.

* Corresponding author, Abdenacer.Makhlouf@uha.fr

2. INDEX OF LIE ALGEBRAS

Throughout this paper $\mathbb{K}$ is an algebraically closed field of characteristic 0. In this Section we summarize the index theory of Lie algebras.

Definition 1. *A Lie algebra $\mathscr{G}$ over $\mathbb{K}$ is a pair consisting of a vector space $\mathbb{V} = \mathscr{G}$ and a skew-symmetric bilinear map $[\ ,\ ] : \mathscr{G} \times \mathscr{G} \to \mathscr{G}$ $(x,y) \to [x,y]$ satisfying the Jacobi identity*

$$[x,[y,z]] + [y,[z,x]] + [z,[x,y]] = 0 \qquad \forall x,y,z \in \mathscr{G}.$$

Let $x \in \mathscr{G}$. We denote by adx the endomorphism of $\mathscr{G}$ defined by $adx(y) = [x,y]$ $\forall y \in \mathscr{G}$.

Let $\mathbb{V}$ be a finite-dimensional vector space over $\mathbb{K}$ provided with the Zariski topology, $\mathscr{G}$ be a Lie algebra and $\mathscr{G}^*$ its dual. Then $\mathscr{G}$ acts on $\mathscr{G}^*$ as follows:

$$\begin{array}{rcl} \mathscr{G} \times \mathscr{G}^* & \to & \mathscr{G}^*, \\ (x,f) & \mapsto & x \cdot f, \end{array}$$

where $\forall y \in \mathscr{G} : (x \cdot f)(y) = f([x,y])$.

Let $f \in \mathscr{G}^*$ and Φ_f be a skew-symmetric bilinear form defined by

$$\begin{array}{rcl} \Phi_f : \mathscr{G} \times \mathscr{G} & \to & \mathbb{K}, \\ (x,y) & \mapsto & \Phi_f(x,y) = f([x,y]). \end{array}$$

We denote the kernel of the map Φ_f by $\mathscr{G}^f$:

$$\mathscr{G}^f = \{x \in \mathscr{G} : f([x,y]) = 0 \ \ \forall y \in \mathscr{G}\}.$$

Definition 2. *The index of Lie algebra $\mathscr{G}$ is the integer $\chi_{\mathscr{G}} = \inf\left\{\dim \mathscr{G}^f ; f \in \mathscr{G}^*\right\}$. A linear functional $f \in \mathscr{G}^*$ is called regular if $\dim \mathscr{G}^f = \chi_{\mathscr{G}}$. The set of all regular linear functionals is denoted by $\mathscr{G}_r^*$.*

Remark 3. The set $\mathscr{G}_r^*$ of all regular linear functionals is a nonempty Zariski open set.

Let $\{x_1,\ldots,x_n\}$ be a basis of $\mathscr{G}$. We can express the index using the matrix $([x_i,x_j])_{1\le i<j\le n}$ as a matrix over the ring $S(\mathscr{G})$, (see [4]). We have the following proposition:

Proposition 4. *The index of an n-dimensional Lie algebra $\mathscr{G}$ is the integer*

$$\chi_{\mathscr{G}} = n - \mathrm{Rank}_{R(\mathscr{G})}\left([x_i,x_j]\right)_{1\le i<j\le n},$$

where $R(\mathscr{G})$ is the quotient field of the symmetric algebra $S(\mathscr{G})$.

Remark 5. The index of an n-dimensional Abelian Lie algebra is n.

Definition 6. *A Lie algebra $\mathscr{G}$ over an algebraically closed field of characteristic 0 is said to be Frobenius if there exists a linear form $f \in \mathscr{G}^*$ such that the bilinear form Φ_f on $\mathscr{G}$ is nondegenerate.*

In [7] the author described all the Frobenius algebraic Lie algebras $\mathscr{G} = R + N$ whose nilpotent radical N is Abelian in the following two cases: the reductive Levi subalgebra R acts on N irreducibly; R is simple. He classified all the algebraic Frobenius algebras up to dimension 6. See also [16–18] for further computations.

3. LIE ALGEBRAS OF DIMENSION $n<5$

In this section we compute the index of n-dimensional Lie algebras with $n<5$. Let $\mathscr{G}$ be an n-dimensional Lie algebra and $\{x_1,x_2,\dots,x_n\}$ be a fixed basis of $\mathbb{V}=\mathscr{G}$.

Any n-dimensional Lie algebra with $n<5$ is isomorphic to one of the following Lie algebras.

Dimension 2

$\mathscr{G}_2^1:[x_1,x_2]=x_2.$

Dimension 3

$\mathscr{G}_3^1:[x_1,x_2]=x_3.$
$\mathscr{G}_3^2:[x_1,x_2]=x_2,\ [x_1,x_3]=\alpha x_3,\ \alpha\neq 0.$
$\mathscr{G}_3^3:[x_1,x_2]=x_2+x_3,\ [x_1,x_3]=x_3.$
$\mathscr{G}_3^4:[x_1,x_3]=-2x_2,\ [x_1,x_3]=-2x_3.$

Dimension 4

$\mathscr{G}_4^1:[x_1,x_2]=x_2,\ [x_1,x_3]=\alpha x_3,\ [x_1,x_4]=(1+\alpha)x_4,\ [x_2,x_3]=x_4.$
$\mathscr{G}_4^2:[x_1,x_2]=x_2+x_3,\ [x_1,x_3]=x_3,\ [x_1,x_4]=2x_4,\ [x_2,x_3]=x_4.$
$\mathscr{G}_4^3:[x_1,x_3]=x_3,\ [x_1,x_4]=x_4,\ [x_2,x_3]=x_4.$
$\mathscr{G}_4^4:[x_1,x_2]=x_2,\ [x_1,x_3]=\alpha x_3,\ [x_1,x_4]=\beta x_3.$
$\mathscr{G}_4^5:[x_1,x_2]=\alpha x_2,\ [x_1,x_3]=x_3+x_4,\ [x_1,x_4]=x_4.$
$\mathscr{G}_4^6:[x_1,x_2]=x_2+x_3,\ [x_1,x_3]=x_3+x_4,\ [x_1,x_4]=x_4.$
$\mathscr{G}_4^7:[x_1,x_2]=x_3,\ [x_1,x_4]=x_4.$
$\mathscr{G}_4^8:[x_1,x_2]=x_3,\ [x_1,x_3]=x_4.$
$\mathscr{G}_4^9:[x_1,x_2]=2x_2,\ [x_1,x_3]=-2x_3.$

The computations of the index using Proposition 4 lead to the following result.

Proposition 7. *The index of n-dimensional Lie algebras with $n<5$ is*

$$\begin{aligned}\chi\left(\mathscr{G}_2^1\right)&=0,\\ \chi\left(\mathscr{G}_3^i\right)&=1\ \textit{for}\ i=1,2,3,4,\\ \chi\left(\mathscr{G}_4^1\right)&=0\ \textit{if}\ \alpha\neq-1\quad \textit{and}\quad \chi\left(\mathscr{G}_4^1\right)=2\ \textit{if}\ \alpha=-1,\\ \chi\left(\mathscr{G}_4^i\right)&=0\ \textit{for}\ i=2,3,\quad \chi\left(\mathscr{G}_4^i\right)=2\ \textit{for}\ i=4,\dots,9.\end{aligned}$$

Proof. By direct computations we obtain:

Index of the 2-dimensional Lie algebra: The corresponding matrix of $\mathscr{G}_2^1$ is $\begin{pmatrix}0 & x_2\\ -x_2 & 0\end{pmatrix}$.

Since its rank is 2, $\chi\left(\mathscr{G}_2^1\right)=0$.

Index of 3-dimensional Lie algebras:

We make the computation for $\mathscr{G}_3^1$. The corresponding matrix is

$$\begin{pmatrix}0 & x_3 & 0\\ -x_3 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

It is of rank 2, then $\chi\left(\mathscr{G}_3^1\right)=1$.

The corresponding matrices of Lie algebras $\mathscr{G}_3^2$, $\mathscr{G}_3^3$, $\mathscr{G}_3^4$ are of rank 2, so the index is equal to 1.

Index of 4-dimensional Lie algebras: We make the computation for $\mathcal{G}_4^1$. The corresponding matrix of $\mathcal{G}_4^1$ is

$$\begin{pmatrix} 0 & x_2 & \alpha x_3 & (1+\alpha)x_4 \\ -x_2 & 0 & x_4 & 0 \\ -\alpha x_3 & -x_4 & 0 & 0 \\ -(1+\alpha)x_4 & 0 & 0 & 0 \end{pmatrix}.$$

The determinant of this matrix is $(1+\alpha)^2 x_4^2$. Then it is of rank 4 if $\alpha \neq -1$. When $\alpha \neq -1$, the matrix is of rank 2. Thus, $\chi\left(\mathcal{G}_4^1\right) = 0$ if $\alpha \neq -1$ and $\chi\left(\mathcal{G}_4^1\right) = 2$ if $\alpha = -1$.

In a similar way we find that the corresponding matrices for the Lie algebras $\mathcal{G}_4^2$, $\mathcal{G}_4^3$ are of rank 4, so their index is equal to 0, and the corresponding matrices for the Lie algebras $\mathcal{G}_4^4, \ldots, \mathcal{G}_4^9$ are of rank 2, so their index is equal to 2. Details of calculations can be found in [1]. □

4. INDEX OF NILPOTENT AND FILIFORM LIE ALGEBRAS

Let $\mathcal{G}$ be a Lie algebra. We set $\mathcal{C}^0\mathcal{G} = \mathcal{G}$ and $\mathcal{C}^k\mathcal{G} = [\mathcal{C}^{k-1}\mathcal{G}, \mathcal{G}]$, for $k > 0$. A Lie algebra $\mathcal{G}$ is said to be nilpotent if there exists an integer p such that $\mathcal{C}^p\mathcal{G} = 0$. The smallest p such that $\mathcal{C}^p\mathcal{G} = 0$ is called the nilindex of $\mathcal{G}$. Then a nilpotent Lie algebra has a natural filtration given by the central descending sequence: $\mathcal{G} = \mathcal{C}^0\mathcal{G} \supseteq \mathcal{C}^1\mathcal{G} \supseteq \cdots \mathcal{C}^{p-1}\mathcal{G} \supseteq \mathcal{C}^p\mathcal{G} = 0$.

We have the following characterization of nilpotent Lie algebras (Engel's theorem).

Theorem 8. *A Lie algebra $\mathcal{G}$ is nilpotent if and only if the operator adx is nilpotent for all x in $\mathcal{G}$.*

Example 9. We consider the generalized Heisenberg algebra, which is a $(2n+1)$-dimensional Lie algebra $\mathcal{G}$ given, with respect to a basis $\{x_1, x_2, \ldots, x_{2n+1}\}$, by the following nontrivial brackets:

$$[x_{2i+1}, x_{2i+2}] = x_{2n+1}; \quad i = 0, \ldots, n-1.$$

The associated matrix of $\mathcal{G}$ is of the form

$$\begin{pmatrix} 0 & x_{2n+1} & \ldots & 0 & 0 & 0 \\ -x_{2n+1} & 0 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & x_{2n+1} & 0 \\ 0 & 0 & \ldots & -x_{2n+1} & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 \end{pmatrix}.$$

This matrix is of rank $2n$, then the index of $\mathcal{G}$ is $\chi(\mathcal{G}) = 1$. The regular vectors are of the form $f = \sum_{i=1}^{2k} g_i x_i^* + x_{2k+1}^*$.

In the study of nilpotent Lie algebras the filiform Lie algebras play an important role. This class was introduced by Vergne [22]. An n-dimensional nilpotent Lie algebra is called *filiform* if its nilindex $p = n-1$. The filiform Lie algebras are the nilpotent algebras with the largest nilindex. If $\mathcal{G}$ is an n-dimensional filiform Lie algebra, we have $\dim \mathcal{C}^i\mathcal{G} = n - i$ for $2 \leq i \leq n$.

Another characterization of filiform Lie algebras uses characteristic sequences $c(\mathcal{G}) = \sup\{c(x) : x \in \mathcal{G} \setminus [\mathcal{G}, \mathcal{G}]\}$, where $c(x)$ is the sequence, in decreasing order, of dimensions of characteristic subspaces of the nilpotent operator *adx*. Thus an n-dimensional nilpotent Lie algebra is filiform if its characteristic sequence is of the form $c(\mathcal{G}) = (n-1, 1)$.

The classification of filiform Lie algebras was given by Vergne ([22]) until dimension 6 and was extended to dimension 11 by several authors (see [2,13,14,19]).

Throughout the classification of n-dimensional Lie algebra $n < 5$, there are only two isomorphic classes of filiform Lie algebras, that is $\mathcal{G}_3^1$ and $\mathcal{G}_4^8$, and their indexes are $\chi\left(\mathcal{G}_3^1\right) = 1$, $\chi\left(\mathcal{G}_4^8\right) = 2$.

The 5-dimensional filiform Lie algebras are isomorphic to one of the following Lie algebras:

$$\mathcal{G}_5^1 : [x_1, x_i] = x_{i+1}, \text{ for } i = 2,3,4,$$
$$\mathcal{G}_5^2 : [x_1, x_i] = x_{i+1}, \text{ for } i = 2,3,4 \text{ and } [x_2, x_3] = x_5.$$

Their indexes are $\chi\left(\mathcal{G}_5^1\right) = 3$, $\chi\left(\mathcal{G}_5^2\right) = 1$. The regular vectors of $\mathcal{G}_5^1$ are of the form $f = g_1x_1^* + g_2x_2^* + g(x_3^* + x_4^* + x_5^*)$ with $g \neq 0$ and the regular vectors of $\mathcal{G}_5^2$ are of the form $f = (\sum_{i=1}^4 g_ix_i^*) + x_5^*$.

In the general case there are two classes L_n and Q_n of filiform Lie algebras which play an important role in the study of the algebraic varieties of filiform and more generally nilpotent Lie algebras.

Let $\{x_1, \ldots, x_n\}$ be a basis of the $\mathbb{K}$ vector space L_n. The Lie algebra structure of L_n is defined by the following nontrivial brackets:

$$[x_1, x_i] = x_{i+1}, \quad i = 2, \ldots, n-1. \tag{1}$$

Let $\{x_1, \ldots, x_{n=2k}\}$ be a basis of the $\mathbb{K}$ vector space Q_n. The Lie algebra structure of Q_n is defined by the following nontrivial brackets:

$$\begin{aligned} [x_1, x_i] &= x_{i+1}, \quad i = 2, \ldots, n-1, \\ [x_i, x_{n-i+1}] &= (-1)^{i+1} x_n, \quad i = 2, \ldots, k, \quad \text{where } n = 2k. \end{aligned} \tag{2}$$

The classification of n-dimensional graded filiform Lie algebras yields two isomorphic classes L_n and Q_n when n is odd and only the Lie algebra L_n when n is even.

It turns out that any filiform Lie algebra is isomorphic to a Lie algebra obtained as a deformation of a Lie algebra L_n.

We aim to compute the indexes of L_n and Q_n and regular vectors.

Let $\{x_1, x_2, \ldots, x_n\}$ be a fixed basis of the vector space $\mathbb{V} = L_n$ (resp. $\mathbb{V} = Q_n$) and $\{x_1^*, \ldots, x_n^*\}$ be a basis of the dual space. Define the Lie algebra L_n (resp. Q_n) with respect to the basis by the brackets (1) (resp. (2)). Set $f = \sum_{i\geq 0} g_ix_i^* \in \mathbb{V}^*$.

Proposition 10. *For $n \geq 3$, the index of the n-dimensional filiform Lie algebra L_n is $\chi(L_n) = n-2$. The regular vectors of L_n are of the form $f = \sum_{i=1}^n g_ix_i^*$ with one of $g_i \neq 0$ where $i \in \{3, \ldots, n\}$.*

Proof. Since the corresponding matrix to the Lie algebra L_n is of the form

$$\begin{pmatrix} 0 & x_3 & \ldots & x_n & 0 \\ -x_3 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -x_n & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix}$$

and its rank is 2, $\chi(L_n) = n-2$. The second assertion is obtained by a direct calculation. □

Proposition 11. *For $n = 2k$ and $k \geq 2$, the index of the n-dimensional filiform Lie algebra Q_n is $\chi(Q_n) = 2$. The regular vectors of Q_n are of the form $f = \sum_{i=1}^n g_ix_i^*$ with $g_n \neq 0$.*

Proof. Since the corresponding matrix to the Lie algebra Q_n is of the form

$$\begin{pmatrix} 0 & x_3 & x_4 & \ldots & x_{n-1} & x_n & 0 \\ -x_3 & 0 & 0 & \ldots & 0 & -x_n & 0 \\ -x_4 & 0 & 0 & \ldots & x_n & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -x_n & \ldots & 0 & 0 & 0 \\ -x_n & x_n & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and its rank is $n-2$, $\chi(Q_n) = 2$. The second assertion is obtained by a direct calculation. □

5. INDEX AND DEFORMATIONS

We study now the evolution by deformation of the index of a Lie algebra. About deformation theory we refer to [9–12] and [15]. Let $\mathbb{V}$ be a $\mathbb{K}$-vector space and $\mathscr{G}_0 = (\mathbb{V}, [\ ,\ ]_0)$ be a Lie algebra. Let $\mathbb{K}[[t]]$ be the power series ring in one variable t and coefficients in $\mathbb{K}$ and $\mathbb{V}[[t]]$ be the set of formal power series whose coefficients are elements of $\mathbb{V}$. A *formal Lie deformation* of $\mathscr{G}_0$ is given by the $\mathbb{K}[[t]]$-bilinear map $[\ ,\ ]_t : \mathbb{V}[[t]] \times \mathbb{V}[[t]] \to \mathbb{V}[[t]]$ of the form $[\ ,\ ]_t = \sum_{i\geq 0}[\ ,\ ]_i t^i$, where each $[\ ,\ ]_i$ is a $\mathbb{K}$-bilinear map $[\ ,\ ]_i : \mathbb{V} \times \mathbb{V} \to \mathbb{V}$, satisfying the skew-symmetry and the Jacobi identity.

Proposition 12. *The index of a Lie algebra decreases by deformation.*

Proof. The rank of the matrix $([X_i, X_j])_{ij}$ increases by deformation, consequently the index decreases. □

Corollary 13. *The index of a filiform Lie algebra is less than or equal to $n-2$.*

Proof. Any filiform Lie algebra $\mathscr{N}$ is obtained as a deformation of the Lie algebra L_n. Since $\chi(L_n) = n-2$ using the previous lemma, one has $\chi(\mathscr{N}) \leq n-2$. □

REFERENCES

1. Adimi, H. *Sur les indices des algèbres de Lie et algèbres associatives*. Master's thesis, Sétif University, 2005.
2. Goze, M. and Ancochea-Bermudez, J.-M. On the varieties of nilpotent Lie algebras of dimension 7 and 8. *J. Pure Appl. Algebra*, 1992, **77**, 131–140.
3. Dergachev, F. and Kirillov, A. Index of Lie algebras of seaweed type. *J. Lie Theory*, 2000, **10**, 331–343.
4. Dixmier, J. *Enveloping Algebras*. Graduate Studies in Mathematics, Vol. 11, AMS, 1996.
5. Elashvili, A. G. *On the index of parabolic subalgebras of semisimple Lie algebras*. Preprint, 1990.
6. Elashvili, A. G. On the index of orisphorical subalgebras of semisimple Lie algebras. *Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR*, 1985, 77, 116–126.
7. Elashvili, A. G. Frobenius Lie algebras. *Funksional. Anal. i Prilozhen*, 1982, **16**, 94–95. English translation: *Functional Anal. Appl.*, 1982, **16**, 4 (1983), 326–328.
8. Elashvili, A. G. Frobenius Lie algebras II. *Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR*, 1985, 77, 127–137.
9. Gerstenhaber, M. On the deformation of rings and algebras. *Ann. Math.*, 1964, **79**, 59–103.
10. Gerstenhaber, M. On the deformation of rings and algebras II. *Ann. Math.*, 1966, **84**, 1–19.
11. Gerstenhaber, M. On the deformation of rings and algebras III. *Ann. Math.*, 1968, **88**, 1–34.
12. Gerstenhaber, M. On the deformation of rings and algebras IV. *Ann. Math.*, 1974, **99**, 257–276.
13. Gòmez, J. R., Jimenéz-Merchán, A., and Khakimdjanov, Y. Low-dimensional filiform Lie algebras. *J. Pure Appl. Algebra*, 1998, **130**, 133–158.
14. Goze, M. and Khakimdjanov, Y. *Nilpotent Lie Algebras*. Kluwer Academic Publishers, MIA 361, 1996.
15. Makhlouf, A. Comparison of deformations and geometric study of associative algebras varieties. *Int. J. Math. Math. Sci.*, 2007, Article ID 18915, 24 pages.
16. Ooms, A. I. On Frobenius algebras. *Commun. Algebra*, 1980, **8**, 13–52.
17. Ooms, A. I. Computing invariants and semi-invariants by means of Frobenius Lie algebras. 2008, arXiv:0806.4178 [math.RT].
18. Raïs, M. and Tauvel, P. Indice et polynômes invariants pour certaines algèbres de Lie. *J. Reine Angew. Math.*, 1992, **425**, 123–140.
19. Seeley, C. 7-dimensional nilpotent Lie algebras. *Trans. Amer. Math. Soc.*, 1993, **335**, 479–496.
20. Tauvel, P. and Yu, R. W. T. Indice et formes linéaires stables dans les algèbres de Lie. *J. Algebra*, 2004, **273**, 507–516.
21. Tauvel, P. and Yu, R. W. T. Sur l'indice de certaines algèbres de Lie. *Ann. Inst. Fourier (Grenoble)*, 2004, **54**, 1793–1810.
22. Vergne, M. Cohomologie des algèbres de Lie nilpotentes. Application á l'étude de la variété des algèbres de Lie nilpotentes. *Bull. Soc. Math. France*, 1970, **98**, 81–116.

Lie algebrate indeksi arvutamine

Hadjer Adimi ja Abdenacer Makhlouf

Töö eesmärgiks on arvutada ja uurida Lie algebrate indeksit. On uuritud n-mõõtmelisi Lie algebraid, kui $n < 5$, teatavate Lie algebrate korral (nn filiform-algebrad), mis moodustavad nilpotentsete Lie algebrate alamklassi. On arvutatud üldistatud Heisenbergi algebrate ja gradueeritud filiform-algebrate indeks. Samuti on uuritud Lie algebrate indeksi evolutsiooni deformatsioonevolutsiooni.