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# Travelling water waves along a quartic bottom profile

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**Abstract.** The problem of transmission of wave energy in strongly inhomogeneous media is discussed with application to long water waves propagating in a basin with a quartic bottom profile. Using the linear shallow-water theory it is shown that the wave component of the flow disturbance is described by a travelling wave solution with an amplitude and phase that vary with distance. This means that the kinetic part of the wave energy propagates over large distances without reflection. Conditions for wave breaking in the nearshore are found from the asymptotic solution of the nonlinear shallow-water theory. Wave runup on a vertical wall is also studied for a quartic bottom profile.

**Key words:** hydrodynamics, travelling waves, shallow water theory, quartic bottom profile, wave breaking, hyperbolic systems, exact solutions, approximated solutions.

## 1. INTRODUCTION

The wave energy transmission over large distances has an evident physical significance. One of the obstructions against energy transmission is associated with local inhomogeneities of medium characteristics along the wave path, which lead to the scattering of the wave energy. Even in 1D geometry this problem is described by a high-order partial differential equation (PDE) with variable coefficients, and the processes of wave transmission and reflection are not easy to analyse. The non-reflecting propagation of monochromatic waves in inhomogeneous media has been studied for certain conditions with applications to different fields: acoustics (Brekhovskikh, 1980), plasma (Ginzburg, 1970), and fluids (Magaard, 1962; Vlasenko, 1987). In all these cases the variations in wave amplitude satisfy the requirement of energy flux conservation, which demonstrates that the waves propagate without a reflection of wave energy. The non-reflecting propagation of water waves of different shape (monochromatic waves, solitary waves, sign-variable pulses, etc.) was recently analysed in detail in (Choi et al., 2008; Didenkulova et al., 2009; Didenkulova and Pelinovsky, 2009) for some specific bottom configurations: a convex bottom profile and U-shaped channels. Mathematically, the physical problem of non-reflecting wave propagation can be linked to the problem of reducing the variable-coefficient PDE to a constant-coefficient PDE (Didenkulova et al., 2008, 2009; Grimshaw et al., 2010). This problem is well studied mathematically using direct methods and Lie algebra (Clements and Rogers, 1975; Bluman and Kumei, 1987; Ibragimov and Rudenko, 2004; Grimshaw et al., 2009). This similarity gives us an opportunity to formulate the Cauchy problem, find Green's function, etc. for the initial variable-coefficient PDE. It was used in (Didenkulova et al., 2009) for analysis of wave regimes in a basin with a convex bottom profile $(h \sim x^{4/3})$.

In this paper we introduce another quartic bottom profile $(h \sim x^4)$ that also admits non-reflecting wave propagation. The paper is organized as follows. Travelling waves along a quartic bottom profile are discussed

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in Section 2 within the framework of the linear shallow-water theory. It is shown that the wave of the flow velocity, which corresponds to the kinetic energy of the wave ($\sim \int u^2 dt$), propagates without inner reflection in this bathymetry and conserves its shape in time. However, the waves of water displacement, which corresponds to the potential energy ($\sim \int \eta^2 dt$) of the wave, do not conserve their shape. Nonlinear regimes of the wave shoaling in the nearshore are studied in Section 3. This section also includes characteristics of the wave breaking, which are determined within the framework of asymptotic solutions of the nonlinear shallow-water theory. The wave interaction with a vertical wall located near the shoreline is discussed in Section 4. It is shown that the water displacement at the wall is defined by the time derivative of an incident wave shape. The main results are summarized in the Conclusion.

## 2. LINEAR TRAVELLING WAVE SOLUTIONS

The basic equations for the description of long waves in a basin of variable depth are the shallow-water equations:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}[h(x)u] = 0, \quad \frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0, \tag{1}$$

where $\eta$ is the water surface displacement, $u$ is the depth-averaged water flow velocity, $h(x)$ is an unperturbed water depth, $g$ is the gravity acceleration, $t$ is time, and $x$ is the coordinate increasing in the offshore direction in the considered problem. The geometry of the problem is presented in Fig. 1.

Eliminating $u$, Eqs (1) can be rewritten in the form of a 1D variable-coefficient wave equation

$$\frac{\partial^2 \eta}{\partial t^2} - g\frac{\partial}{\partial x}\left[h(x)\frac{\partial \eta}{\partial x}\right] = 0. \tag{2}$$

The above equation admits existence of travelling wave solutions, which correspond to the case where the wave propagates along a variable bottom profile without reflection. These solutions can be obtained by reducing Eq. (2) of variable coefficient $h(x)$ to a constant-coefficient wave equation (Didenkulova et al., 2009).

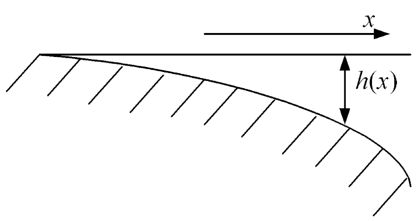


**Fig. 1.** The basic geometry of the system.

Such a transformation exists for the unique bottom profile

$$h(x) = h_0\left(\frac{x}{L}\right)^{4/3}, \tag{3}$$

where $h_0$ is the water depth at some reference location, $x = L$, from the shore. The wave dynamics along the bottom profile of Eq. (3) is studied in detail in (Didenkulova et al., 2009). It is shown that the solution of the Cauchy problem for $\eta$ can be presented as a superposition of two travelling waves propagating in opposite directions but the corresponding velocity field can not be presented in the travelling wave form. From the physical point of view this means that the wave propagates along this specific bottom profile without a reflection of the potential wave energy associated with the water displacement.

Equations (1) can be reduced to a wave equation for $u$ by elimination of $\eta$:

$$\frac{\partial^2 u}{\partial t^2} - g\frac{\partial^2}{\partial x^2}[h(x)u] = 0. \tag{4}$$

It can be shown that Eq. (4) also admits existence of a travelling wave solution $u$ for a specific choice of $h(x)$:

$$h(x) = h_0\left(\frac{x}{L}\right)^{4}. \tag{5}$$

This travelling wave solution for $u(x,t)$ for the bottom profile given by (5) can be interpreted as a wave propagation without reflection of kinetic energy.

In this paper we focus on the travelling wave solutions along the bottom profile of Eq. (5). Similarly to (Didenkulova et al., 2009), this solution can be found in the form

$$u(x,t) = u_0\left(\frac{h}{h_0}\right)^{-3/4} f(t-\tau), \quad \tau(x) = \frac{L(x-L)}{\sqrt{gh_0}\,x}, \tag{6}$$

where $f(\zeta)$ describes the shape of the wave of velocity at $x = L$, $u_0$ is the wave amplitude velocity at the reference location, and $\tau$ is the travel time from $x = L$ to a given point. In our system Eqs (6) describe the wave propagating offshore. The wave propagating onshore should have $f(t+\tau)$ instead of $f(t-\tau)$.

The travelling wave solution (6) is valid for $x \geq 0$, while the travel time $\tau$ varies from minus infinity for $x = 0$ to a fixed value $\tau_0 = L/\sqrt{gh_0}$ for $x \to \infty$. From the physical point of view this means that the wave propagates to the shore over infinite time and formally it never reaches the shore. At the same time, due to the fast amplification of the wave velocity, the wave propagates to the deep water region during a fixed time.

The corresponding solution for the water displacement can be found from the first equation of the system (1):

$$\eta(x,t) = u_0\sqrt{\frac{h_0}{g}}\left(\frac{h}{h_0}\right)^{-1/4} f(t-\tau) - \frac{u_0 h_0}{L} F(t-\tau), \quad (7)$$

where $F(\xi) = \int f(\xi)\,d\xi.$ As above, Eq. (7) corresponds to the wave propagating offshore. The wave propagating to the coast should have the opposite sign before $\tau$ and the second term in Eq. (7).

The solution for the water displacement (7) consists of two terms. The first term is similar to the solution for the velocity (6). Its amplitude decreases with an increase in water depth, while the amplitude of the second term in Eq. (7) does not change with distance. The second term starts to be dominant at great distances from the shore (great water depth).

The solution for the velocity (6) decreases with distance as $\sim h^{-3/4}$, while the water discharge increases as $\sim h^{1/4}$.

It follows from Eq. (7) that in contrast to the wave of velocity (6), the wave of displacement (7) does not conserve its shape (in time) with distance. Hence, we can speak about non-reflecting propagation of kinetic energy only. As a result, the two specific bottom profiles (3) and (5) provide different non-reflecting energy components: potential and kinetic energy.

It is important to mention that the rigorous solution (6) has the same form as the well-known asymptotic solution for smoothly varying depth (Green's law); see for instance (Dingemans, 1996; Mei et al., 2005). The wave propagates slowly near the shore (as indicated above, the travel time to the shoreline tends to infinity), while the duration of the wave in time (wave period) stays limited. Thus, the characteristic time of wave propagation is much greater than the wave period. This proves the applicability of the asymptotic approach to travelling waves at shallow water depths.

At the same time only the first term in Eq. (7) corresponds to this asymptotic solution. Thus, the difference between the rigorous and the asymptotic solution is greater at the greater depth. This surprising result is in contradiction with similar results along the bottom profile (3) (Didenkulova et al., 2009) where the asymptotic solution coincides with the rigorous solution on greater depths. This can be explained by strong and rapid variations of the speed of wave propagation along the quartic profile (5), which breaks the assumptions of the asymptotic approach.

The solution of the Cauchy problem can be obtained and various wave regimes can be analysed using Eqs (6) and (7). It can be seen from these equations that wave amplitudes become significantly large in comparison with the water depth in the nearshore. Hence the nonlinear effects should be taken into account. This is discussed in the next section.

## 3. NONLINEAR DEFORMATION AND WAVE BREAKING

Nonlinear water waves of finite amplitude are described by

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}([h(x)+\eta]u) = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0. \quad (8)$$

Using the procedure described above, the second-order PDE for the flow velocity can be obtained from Eqs (8)

$$\frac{\partial^2 u}{\partial t^2} - g\frac{\partial^2}{\partial x^2}(hu) = g\frac{\partial^2}{\partial x^2}(\eta u) - \frac{1}{2}\frac{\partial^2}{\partial t \partial x}(u^2). \quad (9)$$

Since $\eta$ has not been removed, Eq. (9) is not closed for the function $u(x,t)$. To close it we use the assumption of weak nonlinearity or small-amplitude waves, and use the linear relation between the functions $u(x,t)$ and $\eta(x,t)$ in the right-hand side of Eq. (9). It is evident that nonlinear effects should be essential at shallow depths only, when the first term in Eq. (7) is dominant. As a result, the water displacement $\eta(x,t)$ in the linear approximation can be expressed through the flow velocity $u(x,t)$ (here we consider the wave propagating onshore)

$$\eta \approx -\sqrt{\frac{g}{h(x)}}u, \quad (10)$$

and Eq. (9) can be closed for the flow velocity $u(x,t)$

$$\frac{\partial^2 u}{\partial t^2} - g\frac{\partial^2}{\partial x^2}(hu) = -\frac{\partial^2}{\partial x^2}(\sqrt{gh}u^2) - \frac{1}{2}\frac{\partial^2}{\partial t \partial x}(u^2). \quad (11)$$

Equation (11) is a second-order nonlinear PDE with variable coefficients and it is hard to find travelling wave solutions rigorously. Here we obtain an approximate solution of Eq. (11) for weakly nonlinear waves using the fact that the linear travelling wave solutions (6) and (7) at shallow depths have the same structure as the asymptotic Wentzel–Kramers–Brillouin (WKB) solution, which is obtained for the case of a smoothly varied bottom profile. In the case of weakly nonlinear waves the right-hand side of Eq. (11) is small. Since the bottom slope of the quartic bottom profile is small $(dh/dx = 4h/x \sim x^3)$ in the nearshore, we neglect the terms in the right-hand side of Eq. (11) containing derivatives of the water depth, which are proportional to the bottom slope. In other words, here we use the same assumptions as in the nonlinear asymptotic WKB method (Pelinovsky, 1982; Engelbrecht et al., 1988; Didenkulova, 2009). As a result, we may asymptotically reduce the second-order PDE to a first-order PDE. In the case of a nonlinear shallow-water system, this procedure is described in detail in (Didenkulova, 2009) and is not reproduced here. An approximated evolution equation for the wave of the flow velocity propagating onshore has the following form (Didenkulova, 2009):

$$\frac{\partial u}{\partial t} - \sqrt{gh}\frac{\partial u}{\partial x} = \sqrt{gh}\frac{3u}{2}\frac{\partial u}{\partial x} + \frac{3u}{4}\sqrt{\frac{g}{h}}\frac{dh}{dx}, \qquad (12)$$

where the right-hand side is proportional to a small parameter characterizing the small wave amplitude and low bottom slope. In the first order of this small parameter, the solution of Eq. (12) can be presented as:

$$u(x,t) = u_0\left[\frac{h}{h_0}\right]^{-3/4} f\left[t + \tau - \frac{3u}{2g}\left(\frac{h}{h_0}\right)^{3/4}\int\left(\frac{h}{h_0}\right)^{-3/4}\frac{dx}{h}\right]. \qquad (13)$$

Equation (13) differs from the linear wave (6) by the nonlinear correction in the phase. This solution with various modifications has been obtained earlier for water waves above a gentle beach (Burger, 1967; Varley et al., 1971; Gurtin, 1975; Pelinovsky, 1982; Caputo and Stepanyants, 2003). It describes the nonlinearly deformed Riemann wave of small amplitude. When such a wave approaches the coast, its amplitude increases due to the shoaling effect and its shape becomes more asymmetric (the steepness of the front slope of the wave $\partial\eta/\partial x$ increases).

The time derivative of $u(x,t)$ can be calculated from Eq. (13)

$$\frac{\partial u}{\partial t} = u_0\left(\frac{h_0}{h}\right)^{3/4}\frac{df/d\xi}{1+\dfrac{3u_0 df/d\xi}{2g}Y}, \quad Y = \int_x^L\left(\frac{h_0}{h}\right)^{3/4}\frac{dx}{h}. \qquad (14)$$

In the case of small-amplitude waves the derivatives $\partial u/\partial t$ and $\partial f/\partial\xi$ can be expressed through the local wave steepness, $s(x) = \partial\eta/\partial x$, and initial wave steepness, $s_0 = \partial\eta/\partial x(x = L)$. Thus, using Eqs (10) and (14), Eq. (13) can be re-written in the compact form:

$$s(x) = \left(\frac{h_0}{h}\right)^{3/4}\frac{s_0}{1-\dfrac{3s_0}{2}Y}. \qquad (15)$$

It follows from Eq. (15) that the front-slope steepness $(s > 0)$ increases with distance and tends to infinity when

$$Y_{\mathrm{br}} = \int_{L-X_{\mathrm{br}}}^{L}\left(\frac{h_0}{h}\right)^{3/4}\frac{dx}{h} = \frac{2}{3s_0}. \qquad (16)$$

The condition (16) determines the first wave breaking. The breaking length $X_{\mathrm{br}}$ (the distance the wave propagates before breaking) depends on the initial wave steepness $s_0$ and variations of the bottom profile along the wave path. In the case of a quartic bottom profile the breaking length or the breaking depth $h_{\mathrm{br}} = h(L - X_{\mathrm{br}})$ can be found from Eq. (16):

$$\frac{h_{\mathrm{br}}}{h_0} = \left(\frac{s_0}{s_0 + 4h_0/L}\right)^{1/6} = \left(\frac{s_0}{s_0+\alpha}\right)^{1/6}, \qquad (17)$$

where $\alpha = dh/dx(x = L) = 4h_0/L$ is the bottom slope at the initial location $x = L$.

Thus, in the framework of a weakly nonlinear theory the wave always breaks before reaching the coast due to a very strong shoaling effect. The breaking depth $h_{\mathrm{br}}$ depends on the ratio of the initial front-slope wave steepness $s_0$ and the initial bottom slope $\alpha$ (Fig. 2). In a wide range of this ratio $(0.1–1)$ the breaking depth is almost constant and equal to $(0.7–0.9)h_0$.

## 4. WAVE RUNUP ON A VERTICAL WALL

Here we consider the situation when the wave propagates onshore along the quartic beach profile (5) and then reflects from the vertical wall. It is convenient to re-scale all the variables according to the new geometry shown in Fig. 3. The bottom profile is described by

$$h(x) = h_0\left(1+\frac{x}{L}\right)^4, \qquad (18)$$

and the vertical wall is located at $x = 0$ at the water depth $h = h_0$. We assume that the wall is located outside the breaking zone and that the linear shallow-water theory can be applied for this problem.

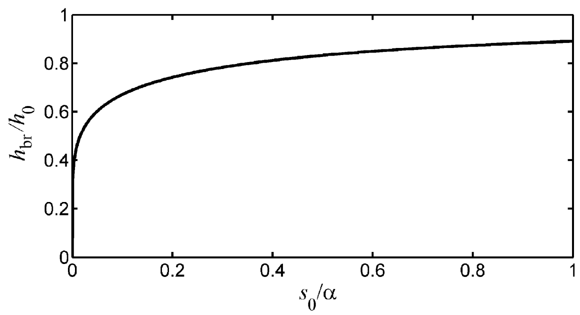


**Fig. 2.** Breaking depth versus wave steepness.

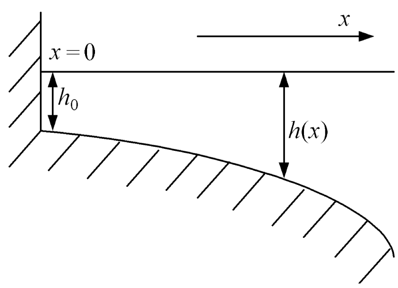


**Fig. 3.** The geometry of wave runup on the wall.

The natural boundary condition at the vertical wall $(x=0)$ is

$$u(x=0,t)=0, \tag{19}$$

which means that the water does not penetrate through the wall. The general solution of the wave equation (4) for a beach profile (5) satisfying the boundary condition (19) can be formulated as the superposition of the incident and reflected waves

$$u(x,t)=u_0\left(1+\frac{x}{L}\right)^{-3}[f(t-\tau+\tau')-f(t+\tau-\tau')],$$
$$\tau(x)=\frac{x}{\sqrt{gh_0}\,(1+x/L)}, \tag{20}$$

where $\tau(x)$ is the travel time from the wall, $\tau'$ is an arbitrary phase shift, and $u_0$ designates the amplitude of the incident wave of the velocity just next to the wall.

The corresponding water displacement can be found from Eq. (7):

$$\eta(x,t)=u_0\sqrt{\frac{h_0}{g}}\left(1+\frac{x}{L}\right)^{-1}[f(t-\tau+\tau')+f(t+\tau-\tau')]$$
$$+\frac{u_0h_0}{L}[F(t+\tau-\tau')-F(t-\tau+\tau')]. \tag{21}$$

The time series of the water displacement and flow velocity at different distances from the vertical wall is shown in Fig. 4 for $\tau'=1.25\tau_0$, where $\tau_0=L/\sqrt{gh_0}$. The figure demonstrates the process of wave shoaling when the wave approaches the vertical wall and the reflection from it. The incident wave has the following initial shape:

$$F(t)=\tau_0\,\mathrm{sech}^2\left(\frac{t}{T}\right), \tag{22}$$

where $T$ is the initial wave duration, which is taken $T=0.1\tau_0$. Far offshore $(x=100L)$ in the deep part the water displacement has the shape of a solitary wave with almost zero flow velocity. It propagates onshore at a very high speed in deep waters. In the nearshore, where the water depth decreases, the wave speed slows down and wave amplitudes of both water displacement and flow velocity grow. At the wall the flow velocity is zero (there is no penetration through the wall), while the water displacement reaches its maximum value. During its propagation the wave also changes its shape, transforming from a solitary bell-shape wave to an *N*-wave.

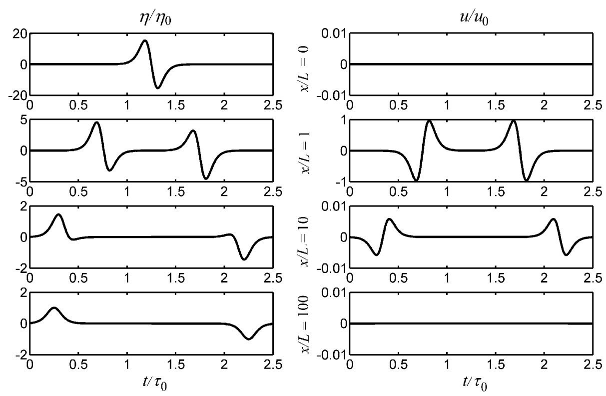


**Fig. 4.** Time series of the sea surface elevations at different distances from the vertical wall for $T/\tau_0=0.1$, $\tau'/\tau_0=1.25$.

The sea surface oscillations at the vertical wall can be calculated from Eq. (21) with respect to the maximum wave height $H_0$ at the infinity

$$R(t)=2H_0\tau_0\frac{dF}{dt}. \tag{23}$$

It should be mentioned that Eq. (23) is universal for all non-reflecting configurations (Choi et al., 2008; Didenkulova et al., 2009). In all these cases the coastal zone “differentiates” the initial wave shape. This differs from the situation on a plane beach where the runup oscillations are determined by the integral transformation of the incident wave shape (Synolakis, 1987). The corresponding maximum runup height with respect to the initial wave height along the non-reflecting bottom configurations is a linear function of the ratio of the travel time to the coast and the wave period. For comparison, in the case of a plane beach the maximum runup height depends on the square root of this ratio. The specific character of each non-reflecting configuration is manifested by the dependence of the travel time on the specific bathymetry.

## 5. CONCLUSION

A new class of travelling wave solutions in a basin of variable depth is studied within the framework of the 1D linear shallow-water theory. It is shown here that the quartic bottom profile $h\sim x^4$ provides the non-reflecting propagation of the wave of flow velocity, while the specific bottom profile $h\sim x^{4/3}$ allows non-reflecting propagation of the wave of the water displacement (Didenkulova et al., 2009). The properties of this new class of travelling waves are discussed. In particular, the wave of displacement does not decay in deep waters and its amplitude tends to be constant. It is shown that travelling waves at small depths are well described by the asymptotic WKB solution. This result differs from the previous cases (in particular, from the bottom profile $h\sim x^{4/3}$) and can be explained by smooth variations of the bottom profile in the nearshore for the studied bathymetry. The nonlinear deformation and breaking of the travelling wave approaching the coast is analysed within the framework of the weakly nonlinear shallow-water theory. The breaking depth depends on the ratio of the initial front-slope wave steepness and the bottom slope. The wave runup on the vertical wall is calculated analytically. The wave amplification and runup are determined by the ratio of the travel time to the coast and the wave period (duration). The obtained results demonstrate the existence of a new class of bottom configurations where the wave energy can be transmitted over larger distances.

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## Lainelevi neljandat järku rannaprofiilidel

Ira Didenkulova ja Efim Pelinovsky

On vaadeldud laineenergia levimist randades, kus vee sügavus kasvab võrdeliselt kauguse neljanda astmega. Madala vee lineaarse teooria raames on näidatud, et sellises keskkonnas levivad kiirusevälja häiritused progressiivsete lainetena, mille amplituud ja faas muutuvad sõltuvalt laineharja asukohast rannaprofiilil. On tõestatud, et laine kineetiline energia ei peegeldu vaadeldavat tüüpi profiililt ja see võib levida suurte vahemaade taha. On leitud taoliste lainete murdumise tingimused vastava mittelineaarse ülesande asümptootilise analüüsi alusel ja on hinnatud selliste lainete maksimaalset uhtekõrgust juhul, kui veepiiril paikneb vertikaalne sein.