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ECONOMICAL DISPATCH OF POWER UNITS UNDER FUZZINESS

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The problem of economical dispatch of condensing power units on the basis of fuzzy information about units' characteristics is considered in the paper. The goal of optimization is minimization of maximum losses caused by uncertainty and fuzziness of the initial information. The paper introduces fuzzy models of characteristics, presents a mathematical model of the problem, the conditions of optimality and the method of solution. Some examples of calculations are presented. Especially important is economical dispatch of power units at coal and oil shale-fired power plants.

Introduction

Economical dispatch of power units in thermal power plants is a primary optimization problem that must be solved for minimization of fuel costs. The methods of economical dispatch based on deterministic initial information are well known [1–3]. These methods require deterministic information about input-output curves of units.

A so-called standard input-output characteristic determined from design calculations or from heat rate tests is usually used for economical load distribution between power units. Standard characteristics are the deterministic function of output power. However, actual characteristics of units are not identical to the standard characteristics. Firstly, standard characteristics include the errors of determination. Secondly, input-output curves of units depend on many other parameters changing during the operation process (quality of fuel, temperature and pressure of steam, temperature of cooling water, and others). Thirdly, the actual characteristics are changing after soiling of units. Fourthly, the characteristics of units also depend on the output power speed. Because of all that the initial information about actual characteristics of units is highly approximate.

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For calculations in real time, the errors of input-output characteristics may be decreased by real-time correcting of standard characteristics. For optimal planning of load distribution it is desirable to consider the information about units in the probabilistic, uncertain or fuzzy form. The initial information in the probabilistic, uncertain or fuzzy form is more complete than deterministic input-output curves, whereas also these forms of information include the information on errors.

The development of probabilistic models for economic dispatch problems began in the 1960s and creation of min-max models in the 1970s [4–7]. During the last years the application of fuzzy information in power-system operation has risen to the centre of attention [8].

In this paper we will examine the fuzzy modeling of unit's input-output characteristics and the models and methods of economical dispatch of power units on the basis of fuzzy information about units' characteristics. The fuzzy form is the most authentic form to look at the information about input-output characteristics of generation units [9].

The main outcome of this paper is development of economical dispatch models for using uncertain and fuzzy information about characteristics and demonstration of the solvability of these problems by the method of planned characteristics [4].

Fuzziness of characteristics

Power generating unit consists of single or two boilers, single turbine and generator (Fig. 1 and 2). Oil shale-fired units have two boilers per unit.

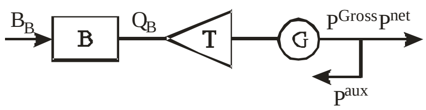

Fig. 1. Power generating unit with a single boiler.

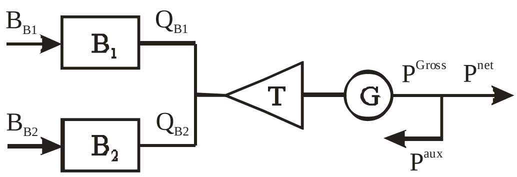

Fig. 2. Power generating unit with two boilers.

B_B – input power (fuel cost) of boiler (MW);
Q_B – output power of boiler (MW);
Q_T – input power of turbine (MW);
P – output power of generator (MW);

P^{Gross} – gross output power (MW);
P^{net} – net output power (MW);
P^{aux} – auxiliary power (MW).

The main characteristics used for modeling boilers, turbine-generators and boiler-turbine-generator units are:

1. Input-output characteristics for:
   boilers: $B_B(Q_B)$,
   turbine-generators: $Q_T(P)$,
   units consisting of boiler, turbine and generator: $B(P)$.
2. Incremental input rate characteristics for:
   boilers: $b_B = \dfrac{\partial B_B}{\partial Q_B}$,
   turbine-generators: $q_T(P) = \dfrac{\partial Q_T}{\partial P}$,
   units consisting of boiler, turbine and generator: $b(P) = \dfrac{\partial B}{\partial P}$.
3. Input rate characteristics for:
   boilers: $\delta_B = \dfrac{B_B}{Q_B}$,
   turbine-generators: $\delta_T(P) = \dfrac{Q_T}{P}$,
   units consisting of boiler, turbine and generator: $\delta(P) = \dfrac{B}{P}$.
4. Efficiency characteristics for:
   boilers: $\eta_B = \dfrac{Q_B}{B_B}$,
   turbine-generators: $\eta_T(P) = \dfrac{P}{Q_T}$,
   units consisting of boiler, turbine and generator: $\eta(P) = \dfrac{P}{B}$.

In order to calculate the efficiency, the input and output powers must be measured in the same units.

A relatively simple uncertain model for above-given characteristics can be obtained if the standard characteristic in the deterministic form will be multiplied to the uncertain multiplier $(1+\hat{w})$, where $\hat{w}$ is the per unit uncertainty rate of the characteristic [9]. Let $B_0(P)$ be a standard input-output characteristic of the unit in the deterministic form. Then the uncertain model for this characteristic is as follows:

$$\hat{B}(P) = B_0(P)\cdot(1+\hat{w}_B), \qquad (1)$$

where

$$w_{B2} \le \hat{w}_B \le w_{B3} \tag{2}$$

w_{B2}, w_{B3} – are the lower and upper rates of uncertainty.

If $B_0(P)$ is a standard input-output characteristic of the power unit, and w_{B2}, w_{B3} are the lower and upper rates of uncertainty, the actual input-output characteristic has the following zone of uncertainty:

$$B^-(P) = B_0(P)(1+w_{B2}) \le B(P) \le B_0(P)(1+w_{B3}) = B^+(P)\,, \tag{3}$$

where

$B^-(P)$ – lower characteristic,

$B^+(P)$ – upper characteristic.

The uncertain zone of the input-output characteristic is shown in Fig. 3.

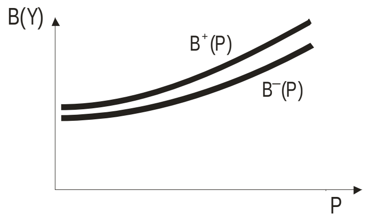

Fig. 3. Uncertain input-output characteristic of the unit.

The zones of uncertainty for others characteristics of units can be modelled analogically. That requires the corresponding standard characteristics and their lower and upper rates w_2, w_3.

In reality the zones of characteristics are not given exactly. Therefore we will look the fuzzy zones of characteristics.

A fuzzy set $\tilde{A}$ is defined in U as a set of ordered pairs [10]

$$\tilde{A} = \left\langle (x, \mu_A(x) \middle| x \in U \right\rangle, \tag{4}$$

where $\mu_A(x)$ is called the membership function, which indicates the degree that *x* belongs to $\tilde{A}$. The membership function takes the values [0, 1] and is defined such that $\mu_A(x) = 1$ if *x* is a member of $\tilde{A}$ and 0 otherwise. At that, if $0 < \mu(x) < 1$, the *x* may be the member of $\tilde{A}$. Situation is undetermined, but *U* is the given crisp set.

Let us assume that the input-output characteristic of the unit has a fuzzy zone, given by the membership function $\mu(w)$. Then the input-output characteristic may be described by the following fuzzy function:

$$\tilde{B}(P) = B_0(P)(1+\tilde{w}_B)\,. \tag{5}$$

Here,

$\tilde{w}_B$ is a fuzzy parameter determined by fuzzy set

$$\hat{W} = \langle W, \mu(w) \rangle , \tag{6}$$

where $\mu(w)$ is the following membership function (Fig. 4):

- if $w_B < w_{B1}$, $\mu(w_B) = 0$
- if $w_{B1} \le w_B \le w_{B2}$, $0 \le \mu(w_B) \le 1$
- if $w_{B2} \le w_B \le w_{B3}$, $\mu(w_B) = 1$
- if $w_{B3} \le w_B \le w_{B4}$, $0 \le \mu(w_B) \le 1$
- if $w_B > w_{B4}$, $\mu(w_B) = 0$.

The zones of fuzzy input-output characteristics are shown in Fig. 4 and membership function in Fig. 5.

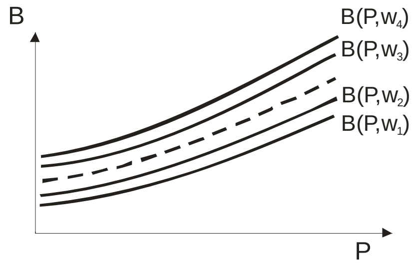

Fig. 4. Zones of fuzzy input-output characteristic.

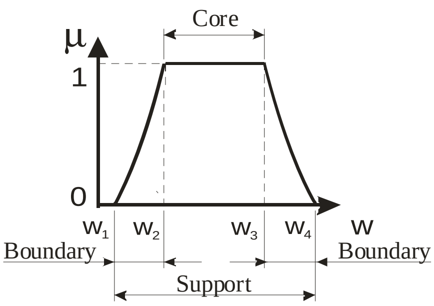

Fig. 5. Membership function.

A fuzzy set consists of three regions (Fig. 5) [11]:

- core, where $\mu = 1$
- support, where $\mu > 0$
- boundaries, where $0 < \mu < 1$.

The fuzzy zones of others characteristics of units can be written by analogy. Using the fuzzy zones it is possible to include subjective information and intuition of specialists in the models of power units.

Deterministic model of economical dispatch

The economical dispatch problem of power units on the basis of deterministic initial information includes minimizing the total fuel cost converted to Euro/h. The problem is usually formulated for one power plant or for a group of power plants. Consider the economic dispatch problem of n power generating units in the following standard form:

$$\underset{P^{Gr}}{Min}\, F(P^{Gr}), \tag{7}$$

where

$$F(P^{Gr}) = \sum_{i}^{n} h_i \cdot B_i(P_i^{Gr}) \tag{8}$$

is subject to

$$P_i^{Gr}(P_i^{net}) = P_i + P_i^{aux}(P_i), \quad i = 1, \ldots, n \tag{9}$$

$$P_D + P_L(P) - \sum_{i=1}^{n} P_i = 0 \tag{10}$$

$$P_{i,\min}^{Gr} \le P_i^{Gr} \le P_{i,\max}^{Gr}, \quad i = 1,\ldots, n, \tag{11}$$

where $B_i(P_i^{Gr})$ – input-output characteristic of unit i (MWh/h);
h_i – fuel price of unit i (Euro/MWh);
P_i^{Gr} – gross power output of unit i;
P – is a vector of gross power outputs;
P_i – net power output of unit i;
$P_i^{aux}(P_i)$ – auxiliary power characteristic of unit i;
$P_{i,\min}^{Gr}, P_{i,\max}^{Gr}$ – generation limits of unit;
P_D – power demand of power plant or power plant group;
$P_L(P)$ – power losses as a function of net outputs of power plants and active loads of network.

Let us carry the equation (8) to the objective function. After that we will have the following objective function:

$$\underset{P}{Min} \sum_{i}^{n} h_i \cdot B_i(P_i^{Gr}(P_i)). \tag{12}$$

The Lagrangian function for the problems (11) and (9) is:

$$\Phi = \sum_{i}^{n} h_i \cdot B_i(P_i^{Gr}(P_i)) + \lambda(P_D + P_L(P) - \sum_{i=1}^{n} P_i), \tag{13}$$

where λ is undetermined Lagrangian multiplier.

The necessary conditions for optimality of the task (11), (9) and (10) are:

$$\frac{h_i \cdot \frac{\partial B_i}{\partial P_i^{Gr}} \cdot \frac{\partial P_i^{Gr}}{\partial P}}{1 - \frac{\partial P_L}{\partial P_i}} = \lambda \text{ , for } P_{i,\min}^{Gr} \le P_i^{Gr} \le P_{i,\max}^{Gr} \text{ , } \quad i = 1,\ldots, n \tag{14}$$

$$\frac{h_i \cdot \frac{\partial B_i}{\partial P_i^{Gr}} \cdot \frac{\partial P_i^{Gr}}{\partial P}}{1 - \frac{\partial P_L}{\partial P_i}} \le \lambda \quad \text{for} \quad P_i^{Gr} = P_{i,\max}^{Gr} \text{ , } \; i = 1,\ldots, n \tag{15}$$

$$\frac{h_i \cdot \frac{\partial B_i}{\partial P_i^{Gr}} \cdot \frac{\partial P_i^{Gr}}{\partial P}}{1 - \frac{\partial P_L}{\partial P_i}} \ge \lambda \quad \text{for} \quad P_i^{Gr} = P_{i,\min}^{Gr} \text{ , } \; i = 1,\ldots, n. \tag{16}$$

When we optimize load distribution in a power plant or between next to each other power plants, the network losses may be neglected.

Economical dispatch under uncertainty

Now we will examine the problem of economical dispatch on the basis of uncertain information about input-output curves of units. Uncertainty of information means that only intervals of characteristics are given. In the given intervals the characteristics are uncertainties. There are several possibilities to optimize loads of units under uncertainty (Laplace, Hurwicz, min-max cost, min-max regret criterions and others [12, 13]. The best criterion for the economical dispatch problem under uncertainty is min-max regret criterion. This criterion is also named a criterion of min-max risk or losses caused by uncertainty of information. Min-max risk criterion guarantees that maximum losses stemming from the uncertainty of information will be as small as possible.

Define the risk function caused by uncertainty of characteristics:

$$R(\overline{P}, W) = F(\overline{P}, W) - F_0(W) \text{ ,} \tag{17}$$

where

$$F(\overline{P}, W) = \sum_{i=1}^{n} h_i \cdot B_i(\overline{P}_i, w_{B,i}) \tag{18}$$

$$F_0(W) = \min_{Pi=1} \sum^{n} h_i \cdot B_i(P_i, w_{B,i}) \text{ .} \tag{19}$$

$F(\overline{P},W)$ is the total cost depending on the planned load distribution of units $\overline{P}$ and on the multipliers of uncertainties *W*.

$F_0(W)$ is the minimum of cost if the load distribution between units is ideally optimal for every *W*.

The economical dispatch problem can now be formulated in the following form:

$$\underset{\overline{P}}{Min}\,\underset{W}{Max}\,R(\overline{P},W)\,, \tag{20}$$

subject to the constraints

$$P_D + P_L(P) - \sum_{i=1}^{n} \overline{P}_i = 0 \tag{21}$$

$$P_{i,\min}^{Gr} \le \overline{P}_i^{Gr} \le P_{i,\max}^{Gr}\,, \quad i = 1,\ldots, n. \tag{22}$$

The conditions of min-max optimality for the task (20)–(22) may be derived on the basis of the main theorem of the game theory with convex functions [14]. The maximizing strategy for the task (20) is the mix strategy, and for minimizing – the pure strategy. The Lagrange function for task (20) and (21) is defined to be the following function:

$$\Psi(\overline{P},\lambda,W) = R(\overline{P},W) + \lambda(P_D + P_L(P) - \sum_{i=1}^{n} P_i)\,. \tag{23}$$

Now we have the problem:

$$\min_{\overline{P}}\,\max_{\lambda}\,\max_{\rho}\,E\Psi(\overline{P},\lambda,W)\,, \tag{24}$$

subject to the constraint (22).

Here ρ is a mix strategy and E is an operator of mathematical expectation.

The main minimizing strategy for the task (24) is as follows:

For a given value of P_D, there exists the value $\lambda_0(P_D)$, such vector of planned power outputs $\overline{P}_0$ and at least n + 1 vertexes of hypercube uncertainty *W* that the following equations are satisfied:

$$\Psi(\overline{P}_0,\lambda_0,W^v) = \Psi(\overline{P}_0,\lambda_0,W^u)\,, \tag{25}$$

where *v* and *u* are the vertexes of zone uncertainty *W*.

The min-max solution guarantees that actual losses stemming from uncertainty of information will be smaller then min-max risk:

$$0 \le R(\overline{P},W) \le \underset{\overline{P}}{Min}\,\underset{W}{Max}\,R(\overline{P},W)\,. \tag{26}$$

In general cases the exact solution of min-max problems is very complicated. In practice the min-max problems can be solved by different approximated methods [4]. The most simple is to solve the min-max problems assuming that $\lambda = const$.

Economical dispatch under fuzziness

Optimization under fuzziness is more general than optimization under uncertainty; in the case of fuzziness it is not possible to guarantee the condition (26). The fuzzy optimization becomes reduced to the min-max optimization, if $\mu = 1$. However, in general the fuzzy optimization is a new kind of optimization.

A typical form of fuzzy incremental input rate characteristic of a power unit is shown in Fig. 6.

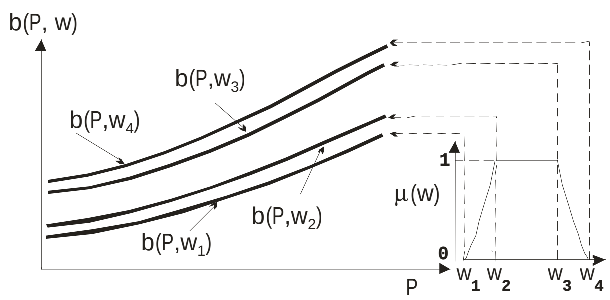

Fig. 6. Fuzzy incremental input rate characteristic of a power unit.

One way to formulate the optimization problems under fuzziness is to reduce these problems to the min-max risk problems. We have studied the following methods of reducing fuzzy optimization problems to the optimization problems under uncertainty:

1) replacement of the fuzzy zones of information to the equivalent crisp zones;
2) formulation of the optimization problems with the fix values of μ, for example: $\mu = 1$; 0.5 and 0.

The first method is the most effective one. In that case the initial membership function in the boundary zones will replace the equivalent membership function of corresponding crisp zones so that (Fig. 7):

$$\int_{w_1}^{w_2} \mu(w)dw = \int_{\hat{w}_1}^{w_2} \hat{\mu}(w)dw\,, \tag{27}$$

and

$$\int_{w_3}^{w_4} \mu(w)dw = \int_{w_3}^{\hat{w}_4} \hat{\mu}(w)dw\,, \tag{28}$$

where

$\mu(w)$ – initial membership function;
$\hat{\mu}(w)$ – reduced membership function.

$$w_2 - \hat{w}_1 = \int_{w_1}^{w_2} \mu(w)dw \quad (29)$$

$$\hat{w}_4 - w_3 = \int_{w_3}^{w_4} \mu(w)dw\,. \quad (30)$$

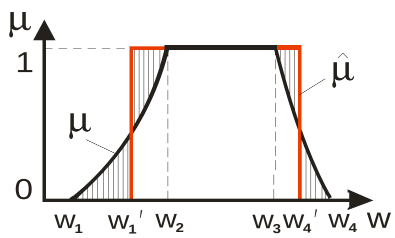

Fig. 7. Initial and reduced membership functions.

The optimization if $\mu = 1$ is too optimistic, and the optimization if $\mu = 0$ is too pessimistic. There exist also others methods of defuzzification [11].

The following steps are performed in sequence to obtain the economic load dispatch between power units for an hour under fuzzy initial information about units' characteristics:

1. Input data:
   1) The standard incremental cost rate characteristics of power units with the inequality constraints;

   $$\hat{b}_i(P_i)\,,\ P_{i,\min} \le P_i \le P_{i,\max}\,,\ i = 1,\ldots, n;$$

   2) The fuzzy multipliers of power units: $w_{i1}, w_{i2}, w_{i3}, w_{i4}$, i = 1,…, n;
   3) The auxiliary characteristics of power units: $P_i^{aux}(P_i)$, i = 1,…, n;
   4) The power demand P_D;
   5) the B matrix loss formula:

   $$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1i}^{n} B_{i0} P_{i} + B_{00}$$

2. Determination of the reduced membership functions of fuzzy multipliers: calculation of the multipliers $\hat{w}_{i1}$ and $\hat{w}_{i4}$, i = 1,…, n.
3. Solution of min-max risk problem with reduced membership functions of fuzzy multiplier.

Practical results

In order to analyse fuzzy optimization efficiency we have made several calculations of economical dispatch using the uncertain and fuzzy information about the input-output characteristics of power units. Some results of these calculations are presented below.

Let us consider the economical dispatch problem between two power units (I and II). Let the power units have the following standard non-linear input-output characteristics:

$$B_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \text{ EUR/h.} \quad (31)$$

The zones of uncertainty and fuzziness are given by fuzzy multiplier w

$$\tilde{B}_i(P_i) = B_i(P_i) \cdot (1 + w_i)\,. \quad (32)$$

The incremental cost rate characteristics are as follows:

$$\tilde{b}_i(P_i) = \frac{\partial B_i}{\partial P_i} \cdot (1 + w_i) = (\beta_i + 2\gamma_i \cdot P_i) \cdot (1 + w_i) \text{ EUR/MWh}$$

$$100 \le P_i \le 200\,,\ i = \text{I, II.} \quad (33)$$

The zones of uncertainty and fuzziness of characteristics are determined by standard characteristics (31) and fuzzy multipliers w_I and w_{II}. The values of coefficients are presented in Table 1.

Table 1. **Polynomial coefficients for power unit's input-output characteristics**

Fuzzy parameter	Notation	α	β	γ
	$B_I(P)$	208.4125	9.6506	0.0058
$w_{B1,I} = -0.05$	$B_{I,min}(P)$	197.9918	9.1680	0.0055
$w_{B2,I} = -0.025$	$B_{I,min}(P)$	203.2022	9.4093	0.0057
$w_{B3,I} = 0.075$	$B_{I,max}(P)$	224.0434	10.3743	0.0062
$w_{B4,I} = 0.15$	$B_{I,max}(P)$	239.6743	11.0981	0.0067
	$B_{II}(P)$	203.2022	9.4093	0.0057
$w_{B1,II} = -0.05$	$B_{II,min}(P)$	193.0420	8.9388	0.0054
$w_{B2,II} = -0.025$	$B_{II,min}(P)$	198.1221	9.1741	0.0055
$w_{B3,II} = 0.075$	$B_{II,max}(P)$	218.4423	10.1150	0.0061
$w_{B4,II} = 0.15$	$B_{II,max}(P)$	233.6825	10.8207	0.0065

The incremental cost rate characteristics of units are shown in Fig. 8 and Fig. 9.

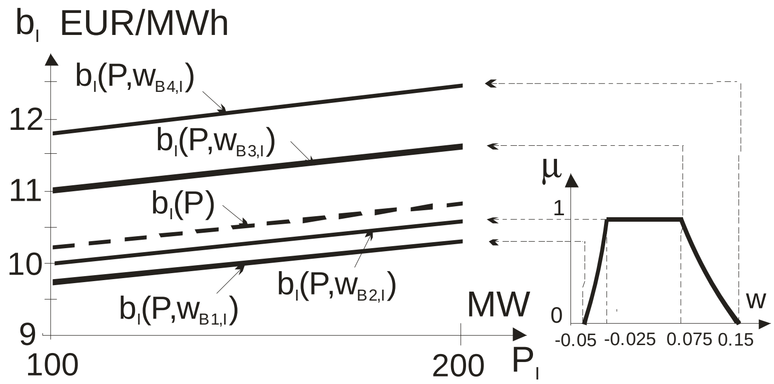

Fig. 8. Incremental cost rate characteristics of unit I.

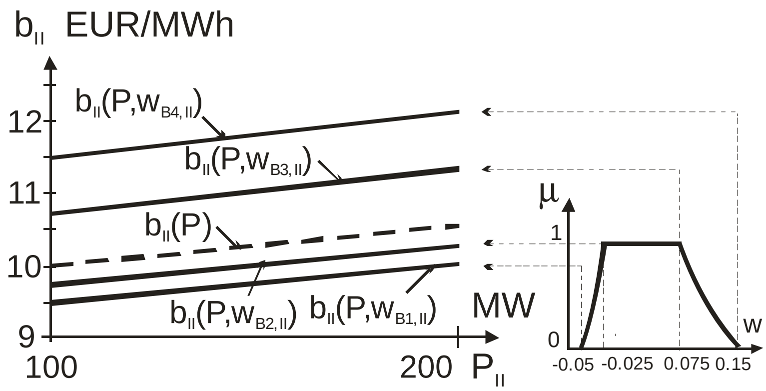

Fig. 9. Incremental cost rate characteristics of unit II.

The optimal generations are obtained using the equations:

$$b_I = b_{II} = \lambda , \tag{34}$$

$$P_D - P_I - P_{II} = 0 , \tag{35}$$

where losses and auxiliary power generations are neglected, and λ is a Lagrange multiplier.

In Table 2 are presented the results of optimization by criterions MaxmaxR and MinmaxR for 3 different cases:

Case A: pure uncertainty ($\mu = 1$), the uncertainty of units' characteristics is given by multiplier values [w_2, w_3].

Case B: expanded uncertainty ($\mu = 0$), the uncertainty of unit's characteristics is given by multiplier values [w_1, w_4].

Case C: considering fuzziness of units' characteristics by the reduced uncertainty method, the uncertainty of unit's characteristics is given by multiplier values [$\hat{w}_1, \hat{w}_4$].

The min-max solutions and risks for cases mentioned above are presented in Tables 2–4.

Table 2. **Min-max solutions and the values of risks for case A** $(\mu = 1)$

P_D, MW	P_I, MW	P_{II}, MW	MinmaxR, EUR/h	MaxmaxR, EUR/h
260	122	138	44	53
280	130	150	55	87
300	138	162	59	111
320	149	171	57	89
340	162	178	43	67

Table 3. **Min-max solutions and the values of risks for case B** $(\mu = 0)$

P_D, MW	P_I, MW	P_{II}, MW	MinmaxR, EUR/h	MaxmaxR, EUR/h
260	126	134	70	140
280	135	145	99	187
300	143	157	132	234
320	155	165	100	191
340	166	174	69	148

Table 4. **Min-max solutions and the values of risks for case C (reduced uncertainty)**

P_D, MW	P_I, MW	P_{II}, MW	MinmaxR, EUR/h	MaxmaxR, EUR/h
260	125	135	57	103
280	133	147	82	141
300	141	159	11	176
320	153	167	82	143
340	164	176	57	110

The diagram of min-max economic loads for cases A, B and C is presented in Fig. 10.

Dependency of risks on the values of μ is shown in Fig. 11.

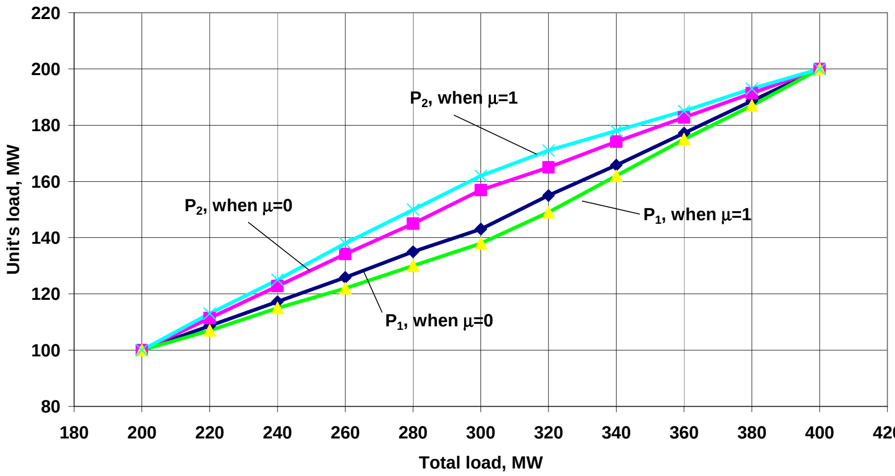

Fig. 10. Diagram of min-max economic loads for Cases A, B, C.

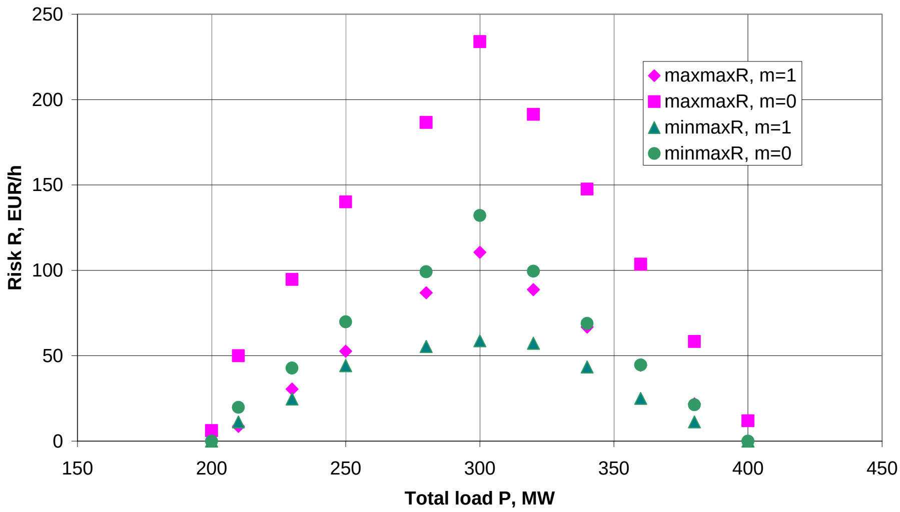

Fig. 11. Dependencies of min-max and max-max risks from the values of μ (m).

Conclusions

1. The methods of economical dispatch under fuzziness must be developed as the most general way of optimization that will be able to consider deterministic, probabilistic, uncertain and fuzzy information.
2. Results of practical calculations show that consideration of uncertain and fuzzy information by the membership functions decreases maximum risk caused by uncertainty and fuzziness in economical

dispatch of power systems approximately 2-3 times. Therefore the studies in this area look promising.

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