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STOCHASTICITY OF THE ELECTRICAL NETWORK LOAD

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Standard deviation of the load changing with time and the expected deviation caused by autocorrelation are observed in this paper. It is proposed to describe the load distribution by so-called peak-normal distribution, which represents a combination of normal distribution, Poisson distribution and lognormal distribution.

Introduction

Regular changes, dependency on weather, operating parameters and stochasticity are characteristic of the electrical network load. Possible load deviations caused by stochasticity must be considered at designing the electrical network and also at planning its operation. It is necessary to know the standard deviation characterizing the level of load stochasticity, but also the distribution enabling to estimate the probability of deviations. The stochastic dependency between random deviations, autocorrelation, is of interest, because it is the basis for short-time forecast.

Modelling of the stochastic component of the electrical network load is done in many papers over the last two decades, and different representations of load distribution and correlation have been made. Its has been shown [1] that most uncertainities of active and reactive daily peak loads in the system can be modelled by normal distributions. Herman and coworkers [2, 3] suggest that the best function to represent low-voltage network load is that of beta distribution of probability. Neimane [4] uses three probability density functions – normal, log-normal and beta distribution – to model variations of the network load (the load measured at the 110-kV level) and concludes that all three distributions provide a reasonably good representation of load variations. However, if variations of the modelled parameter are non-symmetrical, lognormal or beta distribution would give a better approximation.

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Randomness, characteristic of the load, is especially noticeable in the case of smaller loads of the distribution network. Such loads have a rather high standard deviation. In the case of smaller loads, large deviations may occur from time to time, not maching with normal distribution. In this paper the mathematical presentation of the stochastic component considers autocorrelation of load deviation, which is necessary for obtaining short-term load forecast. The peak component, which causes large deviations, is also considered. Large deviations are especially characteristic of distribution network loads. This is one reason why the load is not normally distributed. Due to the above-mentioned facts, it is suggested in this paper that load distribution is a combination of normal, lognormal and Poisson distributions.

Standard deviation of load

The mathematical model describing changes of load (active power, reactive power, or current) consists of three basic components [5]:

$$P(t) = E(t) + \varGamma(t) + \varTheta(t) \ ,$$

where $E(t)$ is mathematical expectation of the load;
$\varGamma(t)$ – temperature-sensitive part of the load;
$\varTheta(t)$ – stochastic component of the load.

Mathematical expectation describes regular changes of a load, such as the general trend and seasonal, weekly, and daily periodicities. Mathematical expectation is principally non-stochastic and corresponds to the normal temperature.

The temperature-sensitive part of a load describes load deviations, caused by deviations of outdoor temperature from the normal temperature. The normal temperature is the average outdoor temperature of the last 30 years on any given hour of the year. If the real outdoor temperature corresponds to the normal temperature, there is no temperature influence. In order to compare the temperature dependencies of different loads, the component $\varGamma(t)$ is normalized:

$$\varGamma(t) = R(t)\gamma(t) \ ,$$

where $R(t)$ is the rate of the temperature dependency of the load,
$\gamma(t)$ – the normalized component of temperature dependency.

The stochastic component $\varTheta(t)$ describes stochastic deviations of the load. Due to autocorrelation, the deviations are stochastically dependent on each other. It is possible to describe the stochastic component of the load by the expected deviation $\zeta(t)$, which represents the conditional mathematical expectation of the stochastic component and normally distributed non-correlated residual deviation (white noise) $\xi(t)$. In addition, it is necessary to consider peak deviations of the load by the component $\pi(t)$, which describes large positive or negative deviations that do not correspond to the normal distribution. It is practical to normalize the stochastic component. The proper rate is the standard deviation of the load $S(t)$. The result is

$$\Theta(t) = S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right].$$

The level of stochasticity is expressed by standard deviation $S(t)$, which changes with time. As an example, standard deviation of weekly and hourly values of the load is presented in Figs. 1 and 2. These examples prove that the changes of standard deviation resemble the changes of load (mathematical expectation), being larger in winter and in the evening and smaller in

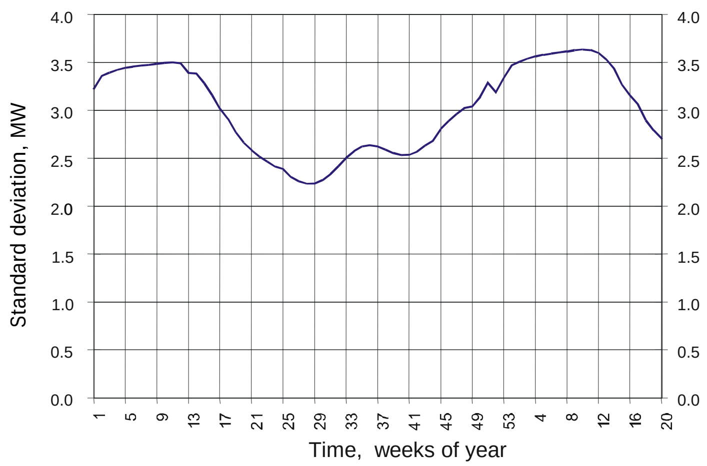

Fig. 1. Load standard deviation, weekly values.

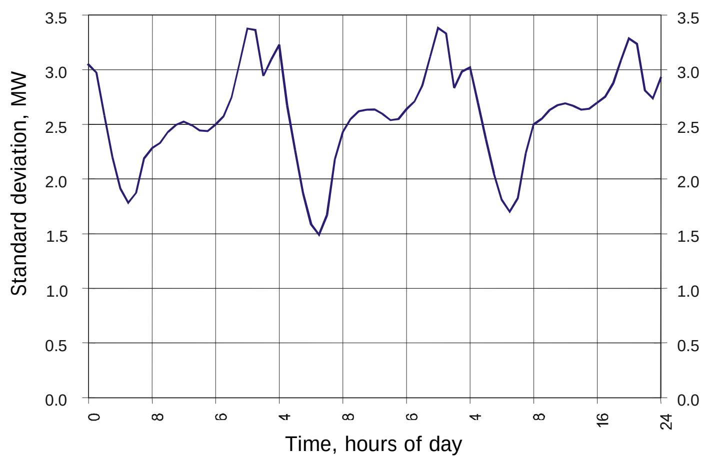

Fig. 2. Load standard deviation, hourly values.

summer and at night. However, a closer observation indicates that changing regularities of standard deviation may not coincide with changes of the mathematical expectation.

Expected deviation of load

Stochastic deviation of the load

$$\vartheta(t) = \frac{1}{S(t)}\left[P(t) - E(t) - \Gamma(t)\right]$$

can be described with the *ARIMA*-model [6] as

$$\vartheta_t = \frac{\Psi(\mathrm{B})}{\Phi(\mathrm{B})}\xi_t \,,$$

where ϑ_t is the value of random deviation in the time interval, $\Phi(\mathrm{B})$ and $\Psi(\mathrm{B})$ are linear operators, and ξ_t is the value of non-correlated time series – residual deviation of the load. Also transfer function may be followed:

$$\mathrm{F(B)} = \frac{\Psi(\mathrm{B})}{\Phi(\mathrm{B})} \,,$$

so that

$$\vartheta_t = \mathrm{F(B)}\xi_\mathrm{t} \,.$$

Actually the operators $\Phi(\mathrm{B})$ and $\Psi(\mathrm{B})$ are presented as

$$\Phi(B) = (1 - \varphi_1 B - \ldots - \varphi_{MF} B^{MF})(1 - \varphi_M B^M)(1 - \varphi_N B^N)$$

and

$$\Psi(B) = (1 - \psi_1 B - \ldots - \psi_{MP} B^{MP})(1 - \psi_M B^M)(1 - \psi_N B^N) \,.$$

Here, the first part of the operators considers the after-effect of load deviations (within the day), which precede the present time interval. The second and third parts of the operator consider the after-effect of one day backward and one week backward. The daily displacement factors *MF* and *MP* are actually within the limits of 1–2, and if the sampling frequency is once an hour, $M = 24$ and $N = 168$. Thus, the model of the stochastic component consists of eight parameters $\varphi_1, \varphi_2, \varphi_{24}, \varphi_{168}, \psi_1, \psi_2, \psi_{24}$ and ψ_{168}.

Large deviations of the load should be excluded, as they do not belong to the residual deviation. The following criterion is suitable:

$$\left|\xi_t\right| < c_S \sigma_\xi \,,$$

where c_S is reliability factor (e.g. 2.7) and σ_ξ – standard deviation of residual deviation. Possible large deviations belong to the load peak component π_t.

Actually, the stochastic component of the load is treated recursively. For each time interval (hour or a part of it), the value of the deviation ζ_t is found by Box-Jenkins model. If the difference $\vartheta_t - \zeta_t$ is suitable according to the previous criterion, $\vartheta_t - \zeta_t$ equals ξ_t, and peak component value $\pi_t = 0$. If not, the peak component value differs from zero, and $\vartheta_t - \zeta_t = \xi_t + \pi_t$. For differentiation between ξ_t and π_t the residual deviation ξ_t is simulated, basing on normal distribution $\xi'_t = N(0, \sigma_\xi)$. Therefore

$$\begin{cases} \xi_t = \vartheta_t - \zeta_t, \quad \pi_t = 0, \quad if\ |\vartheta_t - \zeta_t| < c_S\sigma_\xi \\ \xi_t = \xi'_t, \quad \pi_t = \vartheta_t - \zeta_t - \xi'_t, \quad if\ |\vartheta_t - \zeta_t| >= c_S\sigma_\xi. \end{cases}$$

The results of handling the stochastic deviation ϑ_t are illustrated in Fig. 3.

It is possible to find the value of residual deviation of load ξ_t for each time period (hour) observed. The values of peak deviation of load π_t appear from time to time.

The expected deviation of the load may be used for short-term forecasting of the load. In Fig. 4 an example of the real value of the load and additionally the values of long-term forecast $E(t) + \Gamma(t)$ and short-term forecast $E(t) + \Gamma(t) + S(t)\zeta(t)$ are presented.

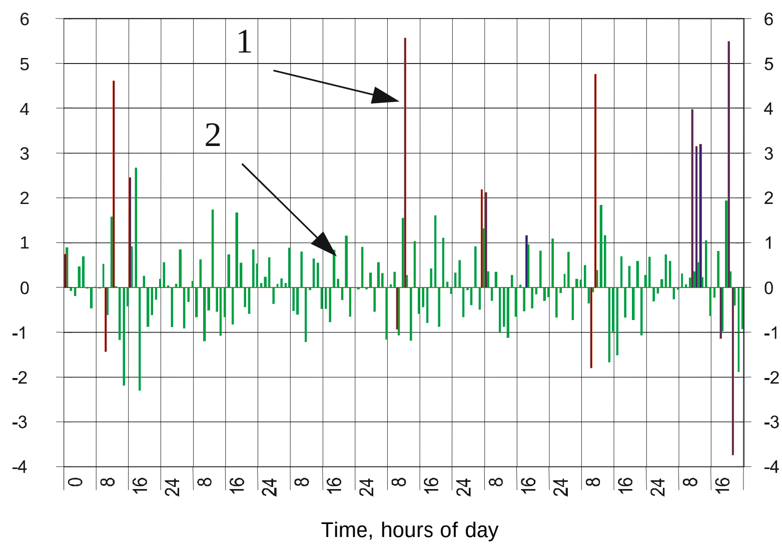

Fig. 3. Peak deviation (1) and residual deviation of load (2).

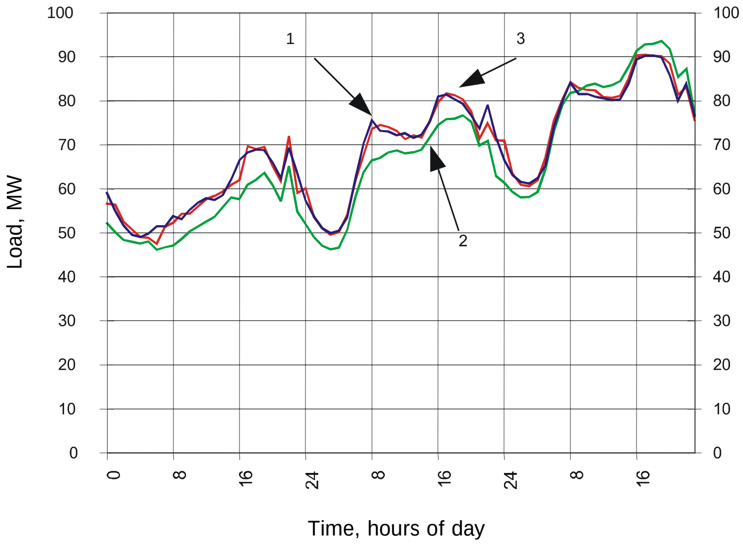

Fig. 4. Real value of the load (1), long-term forecast (2) and short-term forecast (3), hourly values.

Distribution of load

Normal distribution is often considered, but it generally does not apply for electrical network loads. Also the shape of distribution function depends on how the deviation of load ΔP is defined. Attention should be paid to possible load deviation from its mathematical expectation, which is found on the ground of load model as changing with time. If such a model is not used, an average value of load is considered for a longer time period (e.g. a year), and the deviation from that is found. It is possible to found also short-term forecast deviation from conditional mathematical expectation of the load, which considers real progress of load in the recent past and also possible temperature influence. Figures 5 and 6 show deviation histograms, which are found in respect to constant average value of the load (the first case) and in respect to the mathematical expectation, changing with time (the second case). For comparison, normal distribution is also shown in these figures. For assessing maximum load, attention is paid to the "tail" parts of the histograms, which are presented, magnified, in Fig. 7. We can see that the probability of large deviations is considerable, unlike in the case of normal distribution. The more recognisable are the differences, the larger is the given probability.

Residual and peak deviations of the load form together the so-called peak-normal distribution [7]. Let us assume that a random variable X has peak-normal distribution, when among its normally distributed values large deviations appear from time to time – peaks which do not conform to the normal distribution. The frequencies of positive and negative peaks may be different (including zero). Thus the value X at peak-normal distribution consists of a normal component X_0 and a peak component X_Π. The peak component in its turn consists of a positive X_1, negative X_2 and zero component Q:

$$X = X_0 + X_\Pi$$

$$X_\Pi = X_1 + X_2 + Q\ .$$

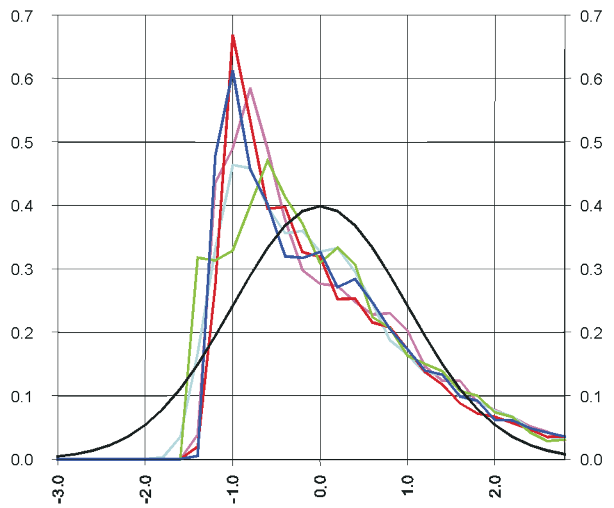

Fig. 5. Histograms of load in the first case.

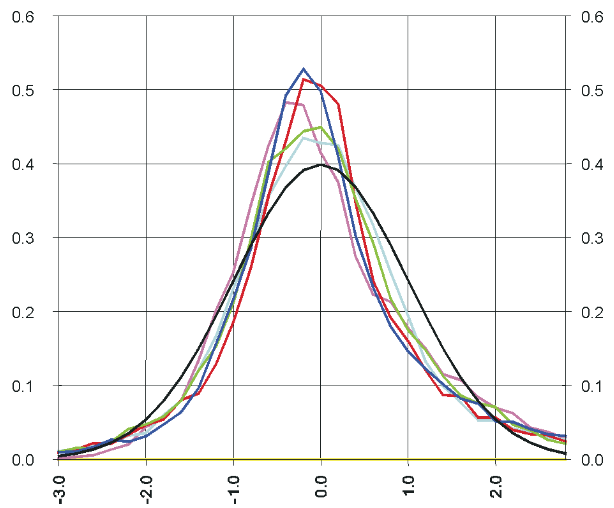

Fig. 6. Histograms of load in the second case.

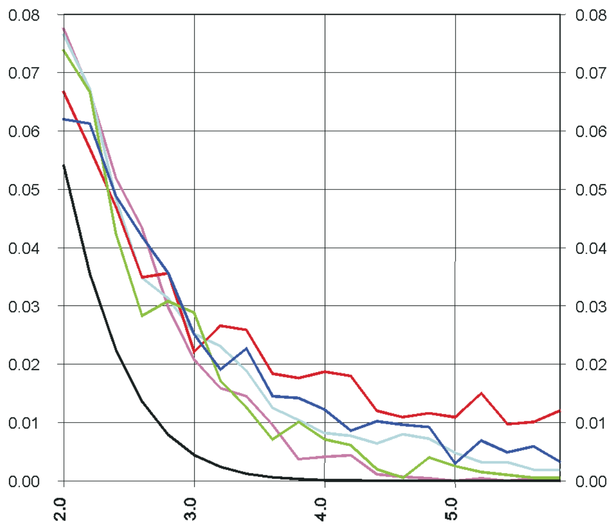

Fig. 7. A fragment of histograms.

Distribution density of the normal component is

$$f_0(x_0) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(x_0-\mu_0)^2}{2\sigma_0^2}} .$$

If frequencies of positive and negative deviations of the peak component are λ_1 and λ_2, respectively, considering that these deviations exclude each other, we get

$$f_\Pi(x_\Pi) = \lambda_1 f_1(x_1) + \lambda_2 f_2(x_2) + \lambda_0 ,$$

where $\lambda_0 = 1 - \lambda_1 - \lambda_2$. Presuming that distribution of deviations is lognormal, we may write

$$f_k(x_k) = \frac{1}{\sqrt{2\pi}\sigma_k |x_k|} e^{-\frac{(\ln|x_k| - \mu_k)^2}{2\sigma_k^2}} ,(k = 1, 2).$$

It is considered that the value of X_2 is negative.

Hence peak-normal distribution is described by 8 parameters: $\mu_0, \sigma_0, \mu_1, \sigma_1, \lambda_1, \mu_2, \sigma_2, \lambda_2$, and it consists of normal, lognormal and Poisson distributions. The last two can be substituted also with some other suitable distributions. Figure 8 illustrates a histogram of the positive peak component and lognormal distribution approximating it.

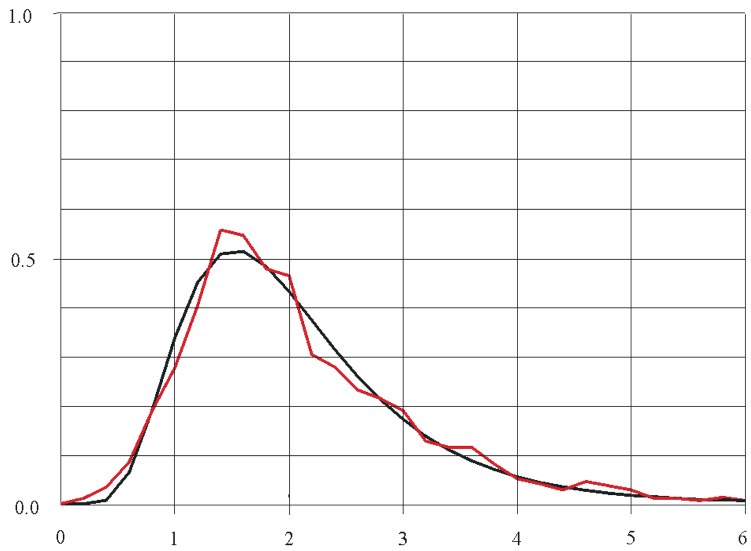

Fig. 8. Histogram and distribution of the positive peak component

Distribution density of the value *X*, which in this case is the load, can be found with convolution

$$f(x) = \int_{-\infty}^{\infty} f_0(x_0) f_\Pi(x - x_0) dx_0.$$

Here f_Π corresponds to distribution density of the peak component. The nature of the value X_0 depends on how the load deviation ΔP is defined. By considering load deviation from the average value $\overline{E}$, mathematical expectation or long- and short-term forecasts of the load, the deviation may be expressed as follows:

1. $\Delta P(t) = E(t) + R(t)\gamma(t) + S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right] - \overline{E}$
2. $\Delta P(t) = R(t)\gamma(t) + S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right]$
3. $\Delta P(t) = S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right]$
4. $\Delta P(t) = S(t)\left[\xi(t) + \pi(t)\right]$

As the values $\gamma(t)$, $\zeta(t)$ and $\xi(t)$ are actually of normal distribution, the equation in the square brackets represents in all cases convolution of the normal distribution with the peak component. The final form of the load distribution will be achieved with linear conversion, which takes into account the deterministic functions *E(t)*, *R(t)* and *S(t)*:

$$f_P(P) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f\left(\frac{P(t) - E(t) - R(t)\gamma(t)}{S(t)}\right) \cdot \frac{1}{S(t)} dt \,,$$

where (t_1, t_2) is the observed time period. Here the functions $E(t)$, $S(t)$ and $R(t)$ are related only to the definition of the first and the second load deviations, in other cases they are missing. Distribution function convolution and linear conversion can both be realised only numerically. Figures 9 and 10 give examples of load distribution, whereby load deviation is found according to the definitions of the first and second cases. Normal distribution density is also given for comparison.

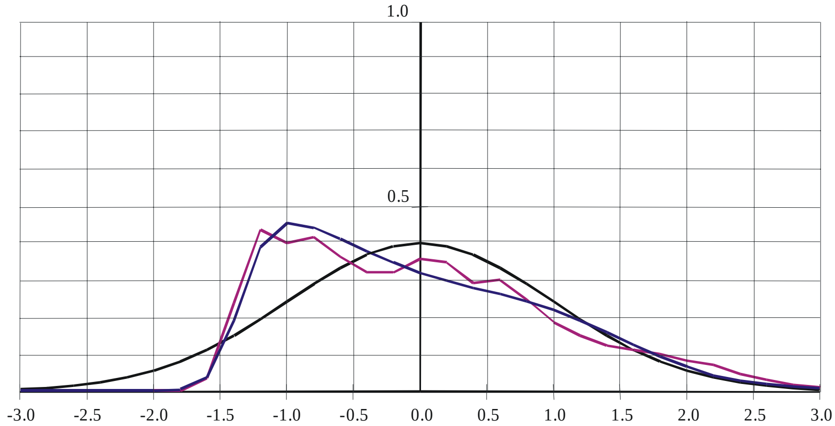

Fig. 9. Load histogram and peak-normal distribution in the first case.

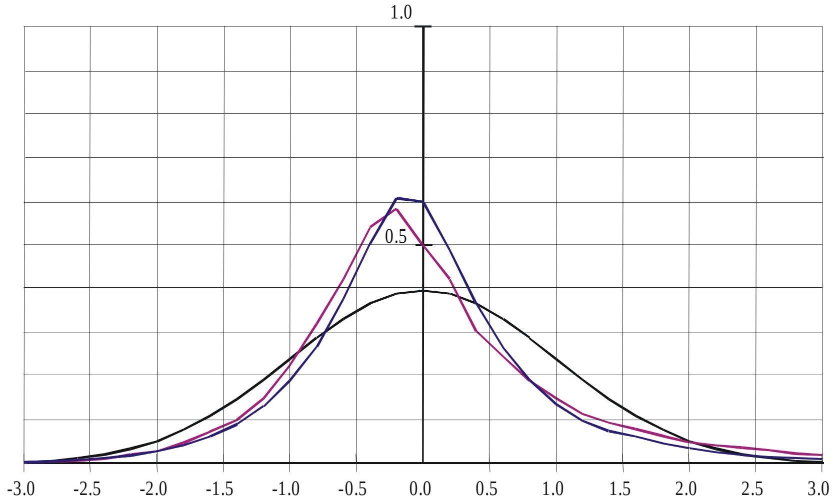

Fig. 10. Load histogram and peak-normal distribution in the second case.

When adding the loads, the sum of normally distributed components is also normal. As a result of convolution, the percentage of the peak component will decrease until it practically disappears. At summation of large number of loads the result will be normal distribution, according to the central-limit theorem of the probability theory. However, the number of addenda is rather big (tens and hundreds). Actually only transmission grid busloads may be considered as normally distributed. This is valid only for relative deviations of the load, presented above in square brackets. If the distribution is found for a longer time period, the distribution will remain asymmetric due to the standard deviation, the rate of temperature dependency and especially because of the change of mathematical expectation. If, for example, the load deviation is considered in respect to the mean value, the load distribution will remain as illustrated in Fig. 5, not depending on the number of summed loads. Anyway, such kind of distribution “tail” is significantly shorter than in the case of the peak component, whereby the assessment of maximum load based on normal distribution does not cause any large errors.

Conclusion

In monitoring the electrical network load, besides mathematical expectation and temperature dependency, stochasticity of the load offers practical interest. It is necessary to evaluate possible stochastic deviations of the load both for short time period, at planning electrical network operation, but also for a longer time period, at electrical network designing. The stochastic dependency caused by autocorrelation enables to find the short-term forecast of the load.

The level of stochasticity is determined by standard deviation, which must be considered as changing with time not only on yearly but also on weekly and daily levels. Expected deviation of the load resulting from autocorrelation can be found (differs from zero) 7–10 days ahead.

Load distribution, depending on determination of load deviation, is needed to evaluate the possible maximal deviations of the load. It is possible to apply the peak-normal distribution representing the combination of normal, lognormal and Poisson distributions.

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