Towards unification of terrestrial gravity data sets in Estonia

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Abstract. Gravity data in Estonia have been collected by different institutions over many decades. This study assesses the suitability of available gravity data for ensuring a 1 cm geoid modelling accuracy over Estonia and in the Baltic Sea region in general. The main focus of this study is on the determination and elimination of discrepancies between three nationwide datasets. It was detected that one tested historic gravity dataset contained inadmissible systematic biases with respect to other tested datasets. Possible ways of gravity data improvement are discussed. More specifically, new field observation campaigns and aspects of using their outcomes in subsequent regional geoid modelling are suggested.

Key words: gravity, Bouguer anomaly, anomaly prediction, geoid, Estonia.

INTRODUCTION AND MOTIVATION OF THE STUDY

Gravity measurements are used for studying the figure, composition, and structure of the Earth. In particular, gravity observations are useful in the computations of geoid models. The geoid is defined as an equipotential surface of the Earth’s gravity field, (generally) inside the topographic masses on land and more or less coinciding with the mean sea level at sea. Due to irregularities in mass distributions within the Earth, the geoidal heights undulate with respect to the reference ellipsoid. However, deviations of the two surfaces do not exceed $\pm$100 m, globally. The geoid plays an essential role in the national geodetic infrastructure as the topographic heights and depths of sea bottoms are determined from it. Thus, many applications in geodesy, geophysics, oceanography, and engineering require physically defined heights related to the geoid.

During the past two decades the need for refined geoid models has increased due to recent technological advancements. For instance, the Global Positioning System (GPS) technology has become a standard tool for solving many tasks, which previously required complicated and time-consuming efforts. In many applications the GPS is being used for the determination of two-dimensional geographical co-ordinates – latitude and longitude. Geodetic heights (reckoned from the global reference ellipsoid!), constituting the third dimension, have mostly been abandoned, because of their non-practical nature, fundamentally different from traditional heights related to the mean sea level. Traditionally, the spirit levelling has been applied to accurate height determination. However, a precise geoid model can be employed to convert the geodetic (GPS-derived) height into a conventional (i.e. sea level-related) height value. This is due to the fact that at discrete points a traditional height could be obtained by algebraically subtracting the value of the geoidal height from the geodetic height. Consequently, for the conversion and combination of these fundamentally different height systems, the geoid model must be known to an accuracy comparable to the accuracy of GPS and traditional levelling, i.e. a few centimetres.

In principle, the geoidal heights can be determined from the global distribution of gravity observations (Stokes 1849). Yet, the application of the Stokes integral formula remains impractical, due to incomplete geographical coverage of the (terrestrial) gravity data. Regional improvements of global geoid models can be obtained by modifications of the original Stokes integral formula, which combines local terrestrial gravity data and the long-wavelength component (i.e. the ‘global trend’) of the geoid. This combination is very useful since some recent advances in technology and geodetic theory have created necessary preconditions for achieving 1 cm accuracy in regional geoid modelling.

The geoid models are strongly dependent on gravity data entering into the solutions. The quality of the gravity data affects directly the quality of subsequent geoid determination, and therefore any systematic or gross errors should be removed from the solution. For instance, the influence of a systematic gravity data error $\sigma_g$ on the geoidal height can roughly be estimated by a simple approximation (cf. Heiskanen & Moritz 1967, eq. 2.234)
where $\sigma_s$ is the resulting uncertainty of the geoidal height, $g$ is the gravity value at the computation point, and $s$ is polar distance. For instance, the presence of a 1 mGal gravity bias within an $s = 100$ km radius around the computation point yields a geoid error in the order of 10 cm. For a discussion on the main factors affecting the geoid modelling accuracy see also Vermeer & Kollo (2007).

The density of the data is also of utmost importance. A geoid model accurate to 1 cm (in relative sense for short baselines below 10 km) can be obtained with gravity data spaced about 2–3 km (see, e.g., Forsberg 2001), provided that all systematic biases are removed from the gravity data. It should be noted that such systematic biases can also influence other practical applications, including the realization of the height system, regional geological mapping, and improvements of metrological standards.

Clearly, perfection of any geoid modelling method is diminished or even meaningless with insufficient data quality and coverage. Therefore a great deal of attention should be paid to the reconciliation of the gravity data. The treatment of the data collected with different methods and equipment, during several decades by different nations and specifications, requires careful study before their use in the geoid computation. Therefore, all undesired systematic biases need to be detected and eliminated, followed by the conversion into the common gravimetric datum.

Gravity surveys are performed also for various geological and geophysical applications, where the main concern is given to the internal precision over the particular area of interests. Therefore the gravity data of such surveys are often collected without rigorous ties to the national gravity network. More specifically, inconsistent reference networks and in some cases even freely selected reference points could be (incorrectly) used as a basis for gravity surveys. In other words, the resulting gravity values of such surveys refer to an arbitrary gravity system.

Accordingly, this contribution focuses on the quality of the gravity data, which are available for the area of our principal interest, Estonia. The emphasis is on assessing the suitability of the data for ensuring a 1 cm geoid modelling accuracy over Estonia and its surroundings. The study is described in seven sections. This introduction is followed by the general characteristics of our study area and used data sources. The Estonian gravity system and the used control points are reviewed in detail. Thereafter relevant aspects of gravity survey data are discussed. Methodology of the tests and the results of statistical analysis are explained. An extended discussion on the results achieved and further improvements conclude the paper.

### STUDY AREA AND DATA SOURCES

The Republic of Estonia lies on the eastern shores of the Baltic Sea, bordering the Republic of Latvia and the Russian Federation. The total area of the Estonian dry land is approximately 45 200 km² (including inner waters and islands), the elevation extremes are 0 m at the shoreline and 318 m in southeastern Estonia. The geographical limits of the study area are from 57.5° to 59.7° northern latitude and from 21.8° to 28.3° eastern longitude (Fig. 1).

Gravity data in Estonia have been collected by different institutions over many decades. Currently, the nationwide gravity network is developed and maintained by the Estonian Land Board (ELB); see the next section for more details. In addition, the ELB has recently carried out several precise gravity surveys. The ELB gravity network and survey points are used as control points in this study. The primary aim of this contribution is to establish a rigorous link between the control points and the following two datasets:

- (i) the 1949–58 gravity survey results by the Institute of Geology of the Estonian Academy of Sciences (IG);

The following four attributes are attached to each gravity point: latitude, longitude, normal height, and gravity value. The accuracy assessment of each data attribute is essential to the present study. The geodetic latitude and longitude are related to the Estonian realization of the new European Terrestrial Reference System ETRS89 (2008). It should be noted that the coordinates of most of the survey points were originally determined in an obsolete coordinate system. For the sake of compatibility with the modern data, the old coordinates were converted into ETRS89 by using some spatial transformation (for more details see the corresponding section below). The heights are referred to the Baltic Height System 1977 (BHS77), i.e. to the mean sea level at the Kronstadt tide gauge. Recall that the Estonian territory is affected by the Fennoscandian post-glacial rebound. Even though the levellings have been performed over a relatively long time span, any temporal changes in the heights were not considered in this study. This is due to the fact that the accuracy of the height component of the gravity survey points is much poorer than the overall range of the land-uplift effect.

The gravity has the physical dimension of acceleration and is often expressed in a CGS unit milligal (1 mGal = 10⁻⁵ m s⁻²). The original gravity observations have been converted to the contemporary Estonian gravity system, which is currently realized through a nationwide set of absolute gravity measurements.

The gravity measured on the Earth’s physical surface is not directly comparable with normal gravity,
which is generated by some equipotential mean Earth ellipsoid. This study adopts the parameters of the international GRS-80 (Moritz 1992) ellipsoid both as a reference for the geodetic coordinates and as a generator of the normal gravity field. The actual gravity observations are reduced to gravity anomalies in such a way that the features under study stand out as correctly as possible. There are several different ways (see, e.g., Heiskanen & Moritz 1967) to represent the measured gravity values. For instance, different types of gravity anomalies are mainly used for (1) determination of the geoid, (2) interpolation and extrapolation of the gravity field, (3) geological mapping and geophysical exploration for natural resources, and (4) investigation of the Earth’s crust and also its deeper layers. In this research the simple Bouguer gravity anomaly is used in comparisons.

CONTROL POINTS IN COMPARISONS

Estonian gravity system and the gravity network points

The first gravity measurements corresponding to international standards were performed in Estonia already in the 1930s within the frame of the activities of the Baltic Geodetic Commission (BGC). A number of gravity reference points were observed with pendulum gravity meters around the Baltic Sea as a result of this cooperation. The Estonian reference point was established in Tallinn by two observing groups (BGC 1937). The observation results were referred to the 1930 realization of the international Potsdam gravity system (hereinafter referred to as PGS1930). In 1938–41 the PGS1930 was extended by using a relative pendulum apparatus for determining over 100 gravity points nationwide. These works were interrupted by the outbreak of World War II. The computations were completed and some results were released only a decade later, in the 1950s. The network was then used as a reference for several gravity surveys, even though the distribution of the points was scarce and their accuracy was even lower (worse than ±1 mGal) than the performance of spring gravity meters used at the time (Maasik 1948).
In 1955–57 new II and III order gravity networks were established in cooperation between the IG and the Institute of the Physics of the Earth (IPE, at the Academy of Sciences of the USSR, Moscow). First, three II order points were established by the IPE on the Tallinn, Kuressaare, and Tartu airfields and thereafter connected to the USSR reference gravity points located in St Petersburg and Moscow (Maasik 1958). Spring gravimeters and most likely an air transport were used for the connection measurements. The three II order points were used for the further densification of the gravity network in Estonia. Altogether 26 points of the III order network were observed by the IG. These II and III order networks can be regarded as the 1955 realization of the gravity works performed in the USSR.

Unfortunately we possess only indirect information about the GUGK gravity campaigns in Estonia, whereas the number and locations of GUGK’s IGSN71 reference points in Estonia still remain ambiguous.

After independence was regained in 1991, the establishment of the Estonian national geodetic networks was initiated. The gravimetric works started with the connection measurements between the known Estonian IGSN71 reference stations (the airfield gravity points in Tallinn, Kuressaare, and Ülenurme, plus a point in the basement of the Institute of Physics at the University of Tartu) and the aforementioned repeated gravity network. In 1992 a solution for the Estonian gravity network was achieved, but the resulting accuracy remained uncertain (Oja 2007). Firstly, the observation details, including the values and accuracy estimations of the old IGSN71 campaigns were not exactly known to Estonian specialists. Secondly, the available, but worn out instrumentation was not capable of performing reliable and precise gravity measurements any more. Their immediate replacement was impossible due to certain financial difficulties at the beginning of the 1990s.

From 1992 the gravity network was improved (both in accuracy and reliability) within the framework of the international co-operation with foreign institutions, such as the Finnish Geodetic Institute (FGI), Danish National Survey and Cadastre, and National Geospatial-Intelligence Agency (NGA) of the USA. As a result the improved solution of the gravity network was introduced in 1999 (Sildvee & Oja 2002). It was based on the determinations of absolute gravity values with a JILAg-5 gravimeter in 1995 as well as relative gravity connections with LCR (LaCoste&Romberg) G-type spring gravimeters. It should be noted that the gravity network of 1999 was more or less consistent with the current realization of the gravity system.

Soon systematic differences were detected in the gravity observation database of the network (see, e.g., Oja & Sildvee 2003). Also, the methods for gravity data reductions and the network adjustment were slightly obsolete (Oja 2005). To improve the observation database, several LCR gravimeters were acquired by the ELB. These were used for high-precision measurements on the gravity network from 2001 onwards. At the same time the development of the measurement methodology and data processing software was started as well. More details about the concepts, the measurements, and the used methods for the modernization of the Estonian gravity network are given by Oja (2008). As a result this network became the current realization of the Estonian gravity system.

The gravity system as a part of the Estonian geodetic system has been officially defined and enforced by the
The gravity network is currently being developed and maintained by the ELB. The Estonian gravity system is realized through the gravity network (named GV-EST95), which is divided into I, II, and III order networks. All information about the network points (coordinates, heights, gravity, etc.) is stored in the ELB geodetic database.

The absolute gravity points form the I order network, which has been observed in three campaigns so far. In 1995 the first modern absolute gravity campaign was carried out in Estonia. Three points of the I order network were observed with a JILAg-5 absolute gravimeter by the FGI. The uncertainty of absolute gravity values was estimated to be ±12 to 13 µGal (at the 95% confidence level) at observation epoch 1995.8 (FGI 2003). In 2007 the measurements at two points were repeated by a team from Institut für Erdmessung (IfE), Leibniz Universität Hannover, Germany. A modern FG-5 absolute gravimeter was used to determine gravity values within ±10 µGal accuracy (L. Timmen, pers. comm. 2008). At the end of 2007 the I order network was expanded by establishing four new absolute gravity points. A year later a FGI team observed both the existing and new I order points (altogether 7) again by using an FG-5 instrument. All these absolute gravity results were processed according to internationally recognized International Absolute Gravity Base Station Network (IAGBN) standards.

In 2001–04 the II and III order gravity network measurements were performed with three LCR G-type relative gravimeters provided by the NGA. In 2004–05 two modern gravimeters Scintrex CG-5 were acquired and used successfully in further gravity works. All used gravimeters have repeatedly been tested and calibrated on the specially designed calibration lines in Estonia and Finland. The 1995 absolute gravity observations served as a basis for the nationwide gravity network adjustment in 2004. The adjusted II order network gravity values were then used for further development of the national gravity networks. By 2006 both II and III order gravity networks (comprising more than 250 points) had been successfully observed (see Fig. 2). Although the

Fig. 2. Distribution of gravimetric control points used in the present study. The total number of control points is 424, comprising 322 national gravity network points and 102 Estonian Land Board precise gravity survey points.
data processing and the adjustment of the network are still in process, our preliminary solutions show that ±50 µGal or even better accuracy has been achieved for II and III order gravity networks. These results have been used in the present study.

The accuracy of the horizontal positions and the heights are within ±30 m and ±10 cm, respectively. It should be emphasized that nearly 200 gravity points coincide with the I and II order geodetic (either GPS or levelling, or both) networks. This provides us with better than ±5 cm accuracy for the positions and heights. Note that over the last decade about 70% of the newer gravity data in Estonia have been positioned by using the GPS technique. The EstGeoid03 (Jürgenson 2003) model has been used to convert the GPS-derived geodetic heights into normal heights. The accuracy of such transformation is estimated to be better than 10 cm. All in all, it can be concluded that the quality of the contemporary gravity network is sufficient for testing the quality of the Estonian gravity surveys.

Gravity surveys of 2004–07

In addition to the gravity network points also results of some recent gravity surveys have been selected in the capacity of control points for the present study. Adjusted II and III order network gravity values serve as a basis for collecting such data. The survey data have been collected in five areas in South Estonia by the ELB (see Fig. 2). In 2006 a number of gravity points along the roads were surveyed with a Scintrex CG-5 gravimeter. In 2007 another set of gravity points was measured along a high-precision levelling line in South Estonia with a CG-5 gravimeter. The measurements were performed in the proximity of 30 levelling benchmarks. During other gravity surveys several points of the GPS densification network (III order geodetic network) were observed in 2004–07. Thus the total number of recent precise gravity survey points exceeds 100. The results of all recent surveys are fully consistent with the national gravity datum. Real-time kinematic GPS measurements in conjunction with the EstGeoid03 model were used for heighting. The accuracy of the survey results was estimated to be better than ±100 µGal for gravity and ±0.1 m for coordinates (including the height). The total number of the control points to be used in the present study is 424, comprising 322 gravity network points and 102 precise gravity survey points (see Fig. 2).

GRAVITY SURVEY DATA

Estonian gravity survey of 1949–58

From 1949 to 1958 some 4200 survey points were observed by the IG. The gravity survey was conducted with different USSR manufactured relative spring gravimeters. The mean standard error of the gravity measurements was claimed to be better than ±0.8 mGal. Barometric levelling was used for the height determination of the survey points. The heighting accuracy is declared to be ±0.5 to ±1.5 m (Sildvee 1998). The present study shows, however, that both error estimates are probably too optimistic. The horizontal coordinates of the survey points were taken from small-scale (1 : 42 000, 1 : 50 000, 1 : 100 000) and often out-of-date topographic maps, therefore the precision of such coordinates reaches only ±50 m at best. The coordinates were related to the Soviet Union Coordinate System 1942. These coordinates were transformed into the new European terrestrial reference system ETRS89 by using a spatial 7-parameter Helmert transformation. The transformation parameters were derived by using collocated accurate historical and contemporary geodetic data (P. Pihlak, pers. comm. 2006).

The survey points (mostly unmarked) are mainly located along the roads with an average distance between stations of about 2–3 km (see Fig. 3). Note that gravity observations are spaced rather irregularly, whereas the roadless regions lack data. The surveying traverses are distributed all over Estonia, producing a density of approximately one survey point per 10 km².

The gravity survey was originally based on the PGS1930-related gravity network. Later, in 1958 these gravity data were reprocessed to relate them to the new (1955) realization of the Potsdam system in Estonia (PGS1955). According to Sildvee (1998), the difference between the PGS1955 and the realization of the modern absolute gravity system in Estonia was about −15.34 mGal in 1995. This correction was introduced to the historic survey results. It should be noted here that in some neighbouring regions a different constant (−14 mGal) was applied in the conversion from the Potsdam system into the IGSN71 (e.g. for the Latvian gravity points, see Kaminskis & Forsberg 1997). Later, the differences between the IGSN71 and the GV-EST95 were found to be −70 to −80 µGal. This can partly be explained by the different treatment of the Earth’s permanent tide in the gravity system. Note that the IGSN71 values are larger than those of the absolute gravity results (Oja 2007).

These historic survey results should be treated with some caution. Several earlier studies (Ellmann 1999; Jürgenson 2003) have revealed that not all individual records are accurate. A number of gross errors were detected and removed during graduate studies (Kaju 2003; Künnapas 2005) at the Estonian University of Life Sciences. As a result the 1949–58 gravity survey database was corrected and updated at the ELB in 2007. The database contains currently 4086 terrestrial gravity observations, which are to be tested in the present study (Fig. 3).
It should be noted that this is the only available set of gravity data covering the whole Estonia with suitable density for geoid modelling. Therefore, this data set has been employed in all the earlier Estonian geoid modelling works (see, e.g., Ellmann 2001, 2004, 2005; Jürgenson 2003).

Gravity survey data from the Geological Survey of Estonia

The GSE initiated gravity surveys for various geological purposes in 1965 (Gromov & Gromova 1968). The main purpose of these surveys was the geological mapping (in 1:50 000 scale) of the crystalline basement. The directions of the gravity survey tracks were selected so that they would cross the main structures of the basement. The average interval between the tracks was 1 km, whereas a usual distance between the stations of the same track was 250 m. Even more detailed special surveys were performed in some spots of geological interest, such as Kärdla meteorite crater, Märgamaa rapakivi pluton, Uljaste uplift, Valgejõe buried valley, etc. The gravity data were also utilized for studying the structure and thickness of the sedimentary cover. This geological mapping programme lasted, with some gaps, from 1970 to 1992 (Gromov & Gromova 1972; Gromov et al. 1981). As a result, the islands of the West Estonian archipelago and most of North Estonia are entirely covered with the GSE gravity survey points. Also some survey areas were mapped in central and southern Estonia (Fig. 4).

The precision of those gravity measurements depended mainly on the development and improvement of the used equipment. More specifically, USSR manufactured gravity meters GNU-KV and GNU-KS and their predecessors were used throughout that period. These quartz (astatic) spring gravity meters are simple, fast operating portable instruments, although without a thermostat. The gravimeters generally possess the nonlinear ‘zero reading’ drift behaviour in time, which requires the establishment of a rather dense local reference network in the survey areas to minimize the drift effect.
In such a network the neighbouring reference stations are located within 5 km, mostly within 2–3 km only.

Different methods were used for determining the gravity values for the local reference network as well. The main method was the forward looping method (also known as a step method). According to this method the measurement bases are tied together in the so-called A-B-A-B sequence. A reading is made at the basepoint A and the instrument is then taken as quickly as possible to the basepoint B. The measurements are then repeated at both basepoints. The time between the readings should be short, which allows us to introduce an assumption on the linearity of the zero drift. Two independent gravity differences are estimated from the four readings (A-B-A-B). If the discrepancies between the two differences are larger than the instrumental precision, the measurements are repeated. The final reading at the basepoint B serves as a basis for further connections (for instance, with the next basepoint C the similar scheme, B-C-B-C, will be executed). For more details see Torge (1989, ch. 6.6.3., p. 251). Also the central point (‘star’) method (A-B-A-C-A-…) was used, but to a lesser extent. Thus the quality of the gravity surveys is also dependent on the adjustment methods of the local survey network. The resulting precision of the local reference networks could be rather heterogeneous in individual survey areas.

Nevertheless, the achieved precision of different gravity surveys was estimated to vary between ±0.1 and ±0.5 mGal, whereas in most cases it was higher than ±0.2 mGal. In general, more recent data are more precise. The heights of the gravity stations were determined by levelling with an average accuracy of ±15 cm. The horizontal coordinates of the survey points were taken from the topographic maps (scales 1 : 25 000 and 1 : 10 000) which used the Soviet Union Coordinate System 1942. These coordinates were transformed into the new European terrestrial reference system ETRS89 in 1997.

Based on the historical technical reports, a digital register of the gravity survey points was started in 1993, together with a major revision of the existing data. The detected errors (typing errors, systematic errors, etc.) were removed. This required also an examination of the gravity systems and realizations used in the past GSE surveys. It was determined that the GI and IPE reference networks were primarily used as a basis for the GSE gravity surveys. Thus the survey results referred to the Estonian realization of PGS1955. After establishing the IGSN71 reference points in Estonia, they were taken as the reference for subsequent surveys. However, as the earlier survey data were stored in paper maps and manuscripts, a decision was made not to introduce the theoretical –14 mGal conversion correction (from

Fig. 4. Distribution of the survey points of the Geological Survey of Estonia (average density: 5 points per km², where available) observed in 1967–2006.
the Potsdam system into the IGSN71 system). Instead, the +14 mGal correction (note the opposite sign!) was introduced to the new survey data in order to keep the consistency with previously collected data, the Bouguer anomaly maps in particular.

In 1967, however, a research unit (enterprise Neftegeofizika) of the Ministry of Geology of the USSR started to develop their own II order networks in the Baltic countries. Within the frame of the extension and new re-adjustment (the IGSN71 system was used) of the USSR II order gravity network some Estonian points were included as well (Azarkina et al. 1984). Starting from 1988 this network was used also in the GSE surveys as a complementary reference network.

In 1991 another Moscow-based research unit (enterprise Spetsgeofizika) completed the establishment of the III order reference network in Estonia (Savchenko 1992). Both networks were used as a basis for the GSE surveys. Basically, the connection with the existing gravity reference networks was mostly a decision made by surveyors. However, since a bias between the two networks was soon detected (All et al. 2002), the preference was given to the II order gravity points, whereas the III order points were seldom used.

For the aforementioned revision the II order reference point No. 389 (Azarkina et al. 1984) located in Tallinn airport was used as the initial point. All the existing GSE datasets were checked with respect to this station. Inspection of this reference point value (belonging to the IGSN71 system) with that of the GV-EST95 revealed a −0.08 mGal difference. The difference was reported but not introduced into the database. Later the discrepancies between the GV-EST95 and Azarkina et al. (1984) levels were checked in other locations in Estonia and were found to be rather similar (on average −0.08±0.02 mGal). The revision determined also some larger biases (in the range of 1.2–1.5 mGal) in several individual regions. Such biases were expected, since the actual 15.4 mGal difference (rather than the theoretical 14 mGal) between the PGS1955 in Estonia and the II order network by Azarkina et al. (1984) was already known at that time. Therefore, in such regions additional field measurements for specifying connections between the GSE data and the national gravity network were made. In some occasions local GSE reference networks were also improved. The revision programme was completed by 2001. Then the gravity value of the Tallinn airport reference point was determined anew with respect to the new nationwide network GV-EST95. Contemporary gravimeters were used and the detected correction (0.08 mGal) was introduced to all GSE data-points as a constant.

As a result a uniform database was created and more or less rigorously related to the current GV-EST95 system. It should be noted that this contribution provides an independent assessment of success of this effort.

The gravity mapping programme at a scale of 1:50 000 was conducted again in 2002–07 (All et al. 2002; All & Gromov 2007), whereas the GV-EST95 system was exclusively used as the reference for all these works. At present (2008) the gravity database of the GSE consists of 126 609 survey points, which are used in the present study.

### METHODOLOGY OF COMPARISONS

Gravity anomalies are used in comparisons and interpolations in this research for detecting possible discrepancies between the datasets in question. This section explains the computation of the anomalies and the anomaly prediction methodology in detail.

#### Computing gravity anomalies

Free-air anomaly, \( \Delta g \), which in a modern context (proposed in Molodenskij 1945) is defined as the difference between the actual gravity measured on the topographic surface \( g(r, \Omega) \) and the normal gravity \( \gamma(r, \Omega) \):

\[
\Delta g(\Omega) = g(r, \Omega) - \gamma(r, \Omega),
\]

where the normal gravity is referred to the surface of the telluroid with the geocentric radius of \( r = r_{GRS} + H \) (\( r_{GRS} \) is the radius of the GRS-80 reference ellipsoid at the latitude of the computation point and \( H \) is the height of the computation point reckoned from the mean sea level) and \( \Omega \) denotes a pair of geodetic coordinates (latitude \( \varphi \) and longitude \( \lambda \)). The normal gravity \( \gamma(r, \Omega) \) on the surface of the telluroid can be approximated as follows (Heiskanen & Moritz 1967, eq. 2-124):

\[
\gamma(r, \Omega) \approx \gamma_0 + \left( \frac{\partial \gamma}{\partial h} \right)_0 H + \frac{1}{2!(\partial^2 \gamma)}_{\Omega} H^2 \\
\approx \gamma_0 - \frac{2\gamma_0}{a} \left[ 1 + f + m + \left( -3f + \frac{5}{2}m \right) \sin^2 \varphi \right] H \\
+ \frac{3\gamma_0}{a^2} H^2,
\]

where the \( ()_0 \) terms denote the vertical gradient of gravity and its first derivative (along the ellipsoidal normal) at the reference ellipsoid. The latitude-dependent normal gravity on the surface of the reference ellipsoid \( \gamma_0 \) can be computed using the equation (cf. Moritz 1992)
\[ \gamma_0 = \gamma_e \frac{1+k\sin^2 \varphi}{\sqrt{1-e^2 \sin^2 \varphi}}. \]  
\hfill (4)

In Eqs (3) and (4) \( a, k, e, m, f \) are geometrical parameters of the adopted reference ellipsoid and \( \gamma_e \) is the normal gravity at the equator. Considering the GRS-80-related constants (Moritz 1992), Eq. (2) can be rewritten as follows:

\[ \Delta g = g(r, \Omega) - \gamma_0 + (7.2125 \times 10^{-3} H + 4.39977 \times 10^{-4} \sin^2 \varphi - 0.3087691)H, \]  
\hfill (5)

where the height \( H \) of the computation point is in metres and the resulting anomaly is in mGal (provided that \( g \) and \( \gamma_0 \) are in mGal as well). The free-air gravity anomalies within Estonian borders vary from -70 to +30 mGal (for a good illustration see Ellmann 2002, fig. 3).

Equation (5) shows that the free-air gravity anomaly is also dependent on height. An error of 3 m in height corresponds approximately to 1 mGal in the anomaly value. Therefore, one has to use accurate heights since any error in heights will propagate to the anomalies. The free-air anomaly is known to be highly correlated with the heights of the topographic masses, so, if there is rough topography in the computation area, the free-air anomalies will be rough as well. Hence the interpolation with free-air anomaly point values could not always be successful. In order to achieve a better result for such an area, the interpolation could be made by means of the less topography-dependent Bouguer anomaly. The Bouguer anomaly is given by the equation

\[ \Delta g_b(\Omega) = g(r, \Omega) - \gamma_0 - 2\pi G \rho H, \]  
\hfill (6)

where the gravitational constant \( G = 6.672585 \times 10^{-11} \) m\(^3\) kg\(^{-1}\) s\(^{-2}\) and \( \rho \) is the density of the topographic masses. In other words, the last term in Eq. (6) removes the attraction of the topographic masses, which are approximated by the Bouguer slab of the height \( H \) with the constant density \( \rho \). Considering the topographic density \( \rho \) conventionally equated to 2670 kg m\(^{-3}\), the last term in Eq. (6) becomes \(-0.11187H\). Hence, the Bouguer anomaly can be calculated via free-air anomaly and height:

\[ \Delta g_b(\Omega) = g(r, \Omega) - \gamma_0 - 0.11187H. \]  
\hfill (7)

These simple Bouguer anomalies will be used in the comparisons (see the following section). It should be noted, however, that the Bouguer slab represents only a very simplified model of topography, since the irregularity of the surrounding topography is entirely neglected. A special term, called terrain correction, can be used for a more realistic accounting of contributions due to mass deficiencies and excesses (with respect to the Bouguer plate; see Heiskanen & Moritz 1967, ch. 3.3). For a rigorous estimation of the effect of topographic masses also the effects of density variations within topography should be taken into consideration. For this the knowledge on the exact 3-dimensional distribution of the topographic density is needed. Unfortunately, this is not known for the whole of Estonia. Therefore, for the sake of simplicity our comparisons neglect both the terrain correction and density variations. The highest discrepancies (but not exceeding 1 mGal) due to this simplification are expected to occur in South Estonian highlands, where the topography is the roughest and the elevations reach up to 318 m.

Strictly speaking, the attraction of the atmospheric masses should also be accounted for in \( g(r, \Omega) \). However, all the datasets in question most likely do not contain the atmospheric correction on the gravity measurements, which is recommended by the IAG (see Moritz 1992, sec. 5). This correction is nearly constant (\(~0.8\) mGal) over a smooth area like Estonia. However, since the atmospheric effect is almost the same for nearby data-points, they are neglected in this study (note that in comparisons they simply would cancel each other out anyway).

**Prediction of gravity anomalies**

The agreement of the surveying datasets (\(\text{(i)}\) and \(\text{(ii)}\), which were listed in the second section) with the control points was determined empirically by the following general scheme. First, the simple Bouguer gravity anomalies are computed at the locations of the gravity survey points. If at least one survey point is located not farther than \(0.1^\circ\) arc-minutes (corresponding to 170 m) from the given control point, its value is compared with the known value of the control point. If no survey points were detected within 170 m, the search radius was duplicated. The values of detected survey points were compared (in the case of multiple survey points a simple arithmetic average was found) with the known value of the control point. In other words, it is reasonable to assume that within this short distance the gravity values should be similar after the attraction of the topographic masses and the height differences have been eliminated.

If there are no survey points in the nearest vicinity (\(\text{i.e.} < 340\) m) of a given control point, the gravity value at each control point is predicted from the closest survey points by interpolation. Interpolation is a critical issue, because any error committed at this stage will directly propagate into the comparison results. A short summary of the applied interpolation procedure is as follows.

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Estonian Journal of Earth Sciences, 2009, **58**, 4, 229–245
Clearly, the interpolation needs to be done over a limited area, since the accuracy of the interpolation decreases dramatically with the distance. In our study the search radius was gradually increased only up to a maximum radius of 6 km (accounted from the given control point). Beyond this distance the gravity anomalies can be considered only weakly correlated, since the local gravity disturbances at a survey point practically have no influence on the locations of the control point in question. Therefore not all the available control points did qualify for the present comparison. They were just too far from the survey points or the configuration of the closest survey points did not meet the pre-defined requirements.

Different methods for interpolation are in use. The present study employed a Matlab’s in-built function GRIDDATA, which is designed to solve an interpolation problem for scattered data points, falling at arbitrary positions in the plane. More specifically it is based on a Delaunay triangulation, which dissects the planar region into a set of non-overlapping triangles, so that any point to be interpolated inside the convex hull of the data lies inside exactly one triangle (Matlab manual, see www.mathworks.com). The default linear interpolant was applied, thus the barycentric coordinates were used to perform linear interpolation within the interpolation triangle.

In our experiment, if no survey points in the near vicinity (i.e. <340 m) of a control point were detected, the search radius was increased gradually (using a 0.1′ step) until at least three survey points formed a triangle in such a way that the control point remained within the boundaries of this surface. These data were then used to interpolate the gravity values at the locations of the control points. Finally the interpolated values \( \Delta g'_b(\Omega) \) were compared with the known values \( g_b(\Omega) \) of the control points. The work of the applied search algorithm is visually explained in Fig. 5.

THE RESULTS OF COMPARISONS

Comparisons with the 1949–58 survey data

A total of 348 control points appeared to be suitable for the comparisons with the 1949–58 survey data. The detected discrepancies between the observed and predicted Bouguer anomalies at the locations of the control points are depicted in Fig. 6.

The distribution of the discrepancies between the two datasets appears to be quite heterogeneous. The largest systematic discrepancies can be observed in two areas in South Estonia; one centred on \(~58.2^\circ\text{N} & ~25.5^\circ\text{E}\) and the other one on \(~57.95^\circ\text{N} & ~26.2^\circ\text{E}\). In these locations an average bias can reach 3–4 mGal, which may cause a local distortion in the geoid model as much as 3 dm (cf. Eq. (1)). This is clearly inadmissible for precise geoid modelling. In the other parts of Estonia, the discrepancies are less, but still worrisomely exceeding 1 mGal. The histogram of the detected discrepancies for Estonia is shown in Fig. 7a.

The histogram shape is clearly right-skewed, which refers to the prevailing negative systematic errors of the old survey dataset over the whole of Estonia. Only some 65% of the detected discrepancies remain within \( \pm 1 \) mGal, which is much worse than initially expected. The standard statistics, root mean square (RMS) error, and minimum and maximum of the comparison are presented in Table 1.

Comparisons with the GSE survey data

As many as 204 control points appeared to be suitable for the comparisons with the GSE survey data. The detected discrepancies between the observed and interpolated Bouguer anomalies at the locations of the control points are depicted in Fig. 8. Full statistics of the comparison can be found in Table 1.

Depending on the region the detected discrepancies are more or less random (<0.5 mGal in most cases). However, there are a few areas, where the GSE dataset
seems to be slightly biased with respect to the control points (see Fig. 8). One such example is the Tartu region centred on ~58.3°N & ~26.8°E. Recall, however, that the GSE surveys are performed for various geological and geophysical applications. More specifically, the GSE gravity survey in the Tartu region was devoted to mapping of ancient buried riverbeds. These structures stretch up to 20 km, whereas their width rarely exceeds 1 km. It has been determined that in this particular region the ancient riverbeds may generate a surplus effect to local anomalies with an amplitude up to 1.2 mGal.

Note also that the terrain roughness is not accounted for in this comparison. Apparently, the terrain correction may also affect the discrepancies, especially at the control points located along the North Estonian (Baltic) klint and at relief extremes in hilly areas (e.g. in South Estonia). The histogram of detected discrepancies is shown in Fig. 7b.

The histogram shows that the discrepancies are normally distributed, whereas nearly 95% of the detected discrepancies remain generally within ±0.5 mGal. The RMS and the mean of the discrepancies are 0.33 mGal and –0.06 mGal, respectively. From the statistical point of view all the discrepancies larger than 1 mGal (three times the RMS error!) can be considered as outliers and therefore need to be excluded from further comparisons. However, we retain them here, since these discrepancies could be caused by the local structures or by the non-rigorous evaluation of the topographic effects. As a matter of fact, all the individual outliers were checked. They were found to be either in the regions with a rapid change in the gravity gradient or in areas of poor coverage. All in all, in spite of a few outliers there is a reasonable agreement between the control values and the GSE gravity survey results.

**SUMMARY AND CONCLUSIONS**

This contribution focuses on the quality assessment of Estonian terrestrial gravity data, which have been collected by different institutions over many decades. The oldest gravity survey points were originally based on the 1930 realization of the international Potsdam gravity system. In the 1950s these (along with newer gravity data) were converted into a new (1955) realization of the Potsdam system in Estonia. In the 1970s the worldwide gravity system IGSN71 was introduced also in Estonia. Further, the contemporary Estonian gravity system is currently based on a nationwide set of absolute gravity measurements. Accordingly, attempts have been made to convert the historic gravity survey results to the current gravity system. However, due to complexity of the problem, the outcome of such conversions may be erratic. The success of these subsequential conversions needs to be checked by an independent dataset. The quality of the two following gravity survey datasets was investigated in this study:

(i) the 1949–58 gravity survey results by the Institute of Geology of the Estonian Academy of Sciences;


The ELB gravity network and recent survey points are used as control points in this study. The total number of the control points used in the comparisons is 424. The accuracy of the control points was estimated to be better than ±100 µGal for gravity and ±0.1 m for coordinates (including the height). Hence, the quality of the control points is sufficient for testing the two gravity surveys. The agreement of the datasets (i) and (ii) with the control points was determined empirically by using the gravity survey results for predicting the simple Bouguer anomalies at the locations of the control points. Recall that the simple Bouguer anomaly field is much smoother than that of the free-air anomaly, therefore the former is more appropriate for the interpolation.

As many as 348 control points were used for the comparisons with the 1949–58 survey data. The detected discrepancies between the observed and predicted Bouguer anomalies at the locations of the control points range from –4.5 to +3.8 mGal, with the RMS of the discrepancies being 1.38 mGal. The mean of the detected discrepancies is –0.53 mGal. Unfortunately, the discrepancies between the two datasets are not random at all. The largest systematic discrepancies can be observed in South Estonia, where an average bias can reach up to 3–4 mGal. Such discrepancies may cause local distortions in the geoid modelling as great as 3 dm (cf. Eq. (1)). This is clearly inadmissible for precise geoid modelling. In some other regions of Estonia the discrepancies are less, however, worrisomely still exceeding ±1 mGal.

A total of 204 control points were used for the comparisons with the GSE survey data. The detected discrepancies between the observed and predicted Bouguer anomalies at the locations of the control points range from –1.9 to +1.9 mGal, with a mean of –0.06 mGal. The RMS of the discrepancies is 0.33 mGal. The detected discrepancies are more or less random, whereas nearly 95% of these remain generally within ±0.5 mGal. All in all, in spite of a few outliers a reasonable agreement between the control values and the GSE gravity survey results was achieved. Note also the mean of the discrepancies is almost zero (~0.06 mGal).

Recall that the emphasis of this study was given to assessing the suitability of the existing gravity data to ensure a 1 cm geoid modelling accuracy within the target area. The accuracy of the GSE gravity data seems
Fig. 6. The detected discrepancies between the observed and interpolated Bouguer anomalies at the locations of the control points. The 1949–58 gravity survey data have been used for the interpolation. The discrepancies \( (\Delta g_5(\Omega) - \Delta g_5'(\Omega)) \) range from \(-4.5\) to \(+3.8\) mGal, with a mean of \(-0.53\) mGal. The RMS of the discrepancies is \(1.38\) mGal. The colours are proportional to the range of the detected discrepancies (cf. the colourbar). Black dots denote the locations of the control points.

Fig. 7. Histograms of detected discrepancies between the observed and interpolated Bouguer anomalies \( (\Delta g_5(\Omega) - \Delta g_5'(\Omega)) \) at the locations of the control points. The vertical axis indicates the frequency of points. (a) The 1949–58 gravity data have been used for the prediction. The total number of the used control points is 348. (b) The gravity survey data of the Geological Survey of Estonia have been used for the interpolation. The total number of the used control points is 204.
Table 1. The results of comparisons

<table>
<thead>
<tr>
<th>Type of comparison</th>
<th>Statistics, mGal</th>
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<tr>
<td></td>
<td>Number of points</td>
</tr>
<tr>
<td>$\Delta g_B (\Omega) - \Delta g_B' (\Omega)$</td>
<td>204</td>
</tr>
<tr>
<td>Control – GSE survey</td>
<td>348</td>
</tr>
<tr>
<td>Control* – EGM08</td>
<td>424</td>
</tr>
</tbody>
</table>

* An average atmospheric correction +0.87 mGal has been removed from the results of the original comparison (Ellmann et al. 2009, table 1).

Fig. 8. The detected discrepancies between the observed and interpolated Bouguer anomalies at the locations of the control points. The gravity data of the Geological Survey of Estonia have been used for the interpolation. The discrepancies ($\Delta g_B (\Omega) - \Delta g_B' (\Omega)$) range from −1.9 to +1.9 mGal, with a mean of −0.06 mGal. The RMS of the discrepancies is 0.33 mGal. The colours of the dots are proportional to the range of the detected discrepancies (cf. the colourbar). The large black dots denote the locations, where the absolute range of differences exceeds 1.0 mGal.
to meet this requirement in the data availability areas. Unfortunately, the GSE surveys do not cover the whole of Estonia. Naturally, within the GSE survey areas the 1949–58 gravity survey data can be downweighted or simply removed from the geoid computations. Over the rest of Estonia, however, the usage of the 1949–58 survey results is still unavoidable as this is the only available set of gravity data covering the whole Estonia with suitable density for geoid modelling.

A natural question arises here – is there any way for correcting the 1949–58 survey data? A general answer to this question is affirmative. For instance, offsets between two smooth surfaces (e.g. a gravity field model formed by the control points and that of the survey data) can be minimized by introducing some polynomial fit (see, e.g., Heiskanen & Moritz 1967, ch. 5–9). The set of the detected discrepancies at the locations of the control points can be used for defining the transformation parameters between the pairs of gravity field models. Thereafter, these parameters can be applied in fitting the survey data model to the gravity values of the control points. Note that this kind of fit is able to remove efficiently the long wavelength discrepancies only. In other words, this method is feasible only when the difference between the two surfaces is more or less constant or one is tilted with respect to another. Unfortunately, this is not the case with the 1949–58 survey data. Recall that the detected discrepancies have large regional extremas (see Fig. 6). This diminishes the usefulness of the polynomial fit algorithm for the 1949–58 survey data.

The fitting of the gravity field models could be developed further, e.g. by constraining control points and adjusting the surface of the survey data model between the control points. However, in our case most likely this rather advanced algorithm would fail as well. Recall that the detected discrepancies are irregular and the distribution of the existing control points is rather scarce. The resulting surface would not be ‘seamless’ and the relations between the datasets would still remain dubious. Alternatively, more advanced surface modelling techniques, such as the collocation (for more details see Moritz 1980) or Kriging (both methods treat the surface as a stochastic signal), can also be tested in future studies.

Another alternative would be to fill the data gaps by using gravity values from global geopotential models. For instance, a new high-resolution Earth gravitational model EGM08 (Pavlis et al. 2008) was released to the public in 2008. The EGM08 takes advantage of recent satellite, terrestrial gravity, elevation and altimetry data. This activity is conducted by the NGA. The resolution of the EGM08 is 5° (corresponding to 9 km, i.e. to the spectral degree of ca 2160). In other words, the spatial resolution of the EGM08 is quite comparable with the density of the 1949–58 survey points.

The performance of the EGM08 model over the Baltic countries was evaluated by Ellmann et al. (2009). One of the main conclusions of their study was that the EGM08-derived gravity quantities agree reasonably well with the terrestrial survey data in the Baltic Sea region, including Estonia. Apparently, the 1949–58 gravity survey data have entirely been utilized in the compilation of the EGM08. This data set has been made available to several international gravity databases since the 1990s. Conversely, the modern gravity network points and the results of new surveys were not accessible at the EGM08 compilation. For detecting the discrepancies between the absolute gravity datum and the EGM08-derived gravity field, the free-air anomalies were computed at the locations of the 424 control points (the same as used in this study) in Estonia. The detected discrepancies between the newly measured and EGM08-derived gravity anomalies (control points minus EGM08) vary from –6.8 to +5.6 mGal, with a mean of –0.50 mGal (see Table 1). This shows also that there are some systematic biases between the control points and that of the EGM08. Note that the mean of the discrepancies (–0.50 mGal) resembles the mean of the discrepancies between the control points and the 1949–58 survey data (–0.53 mGal). Also the distribution of the discrepancies is rather similar (cf. Fig. 6 and Ellmann et al. 2009, fig. 10). In other words, the 1949–58 survey data errors have been absorbed into the EGM08 high-frequency spectrum. Therefore the use of the EGM08 model cannot provide better results than the 1949–58 survey data.

All in all, it seems that for accurate geoid modelling the 1949–58 gravity survey results need to be replaced by new field measurements. Certainly, this is quite a burdensome task, requiring much effort and well co-ordinated actions. However, this is needed for the sake of the consistency of the national gravity datasets. Certainly, over large areas it would be feasible to use airborne gravity surveys, similar to the international Baltic Sea airborne gravity campaign in 1999 (Forsberg et al. 2001) and several others in different parts of the world (see, e.g., Forsberg et al. 2007; Hwang et al. 2007). Due to lack of the local expertise and, most importantly, non-availability of suitable equipment in Estonia, such an airborne campaign does not seem realistic within next few years. Therefore, at present, data improvements can only be achieved with conventional gravity surveys. For this, primary attention should be paid to the most critical regions, which can be identified from the present study: in particular, an area with geographical boundaries from 58.0° to 58.5° northern latitude and from 25° to 26° eastern longitude, and also...
a smaller area with geographical boundaries from 57.8° to 58° northern latitude and from 26° to 26.5° eastern longitude. Additionally, the gravity field over major water bodies, such as Lake Peipsi and Lake Võrtsjärv, also Riga Bay, need to be specified. Our further studies will be devoted to achieving this goal. The detected discrepancies will be studied and ultimately resolved in future gravity field and geoid modelling works.

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Terrestriliste raskuskiirenduse andmete ühtlustamine Eestis

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